(ISSN: 2587-0238)

Cansız Aktaş, M. (2022). A Comparison of Solution Strategies for Proportional and Non-Proportional Problems of Students at Different Education Levels: A Cross-Sectional Study, *International Journal of Education Technology and Scientific Researches, 7(18), 1064-1082.*

DOI: http://dx.doi.org/10.35826/ijetsar.484

Article Type (Makale Türü): Research Article

A COMPARISON OF SOLUTION STRATEGIES FOR PROPORTIONAL AND NON-PROPORTIONAL PROBLEMS OF STUDENTS AT DIFFERENT EDUCATION LEVELS: A CROSS-SECTIONAL STUDY

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Received: 06.03.2022 *Accepted: 20.05.2022* *Published: 15.06.2022*

ABSTRACT

This study aims to examine the strategies students at different education levels (7th-12th grade) use to examine to solve proportional and non-proportional problems. The study uses a cross-sectional research design. The study group consists of 49 middle school students, 24 of whom are in 7th and 25 of whom are in 8th grade, and 130 high school students, 38 of whom are 9th, 33 of whom are 10th, 32 of whom are 11th, and 27 of whom are 12th grade. The study uses a measuring tool created in line with the literature and it contains two problems for each of the four different problem groups, which are missing value, numerical comparison, qualitative reasoning, and non-proportional problems for the data collection. The findings of the study indicate that there were no notable differences between grade levels in the strategies used in missing value and numerical comparison problems, and in qualitative reasoning problems, high school students resorted to more appropriate strategies than middle school students. The study found that 7th, 8th, and 9th-grade students for non-proportional problems that contain a constant relationship and the majority of students from all grade levels for problems that contained an additive relationship made wrong interpretations through multiplicative thinking.

Keywords: Proportional reasoning, proportional problem, non-proportional problem, cross sectional study

INTRODUCTION

Proportional reasoning based on a multiplicative relationship is indispensable for students to make sense of concepts such as fractions, decimal numbers, proportions, and percentages (Sowder et al., 1998). The National Council of Teachers of Mathematics (1989) states that the ability to conduct proportional reasoning starts to develop between the 5th and 8th grade levels. Similarly, the Common Core State Standards for Mathematics (2010) views proportional reasoning as a key area that requires focus. People view proportional reasoning as a significant skill because it is a prerequisite for understanding advanced mathematics subjects that one will encounter in high school and later education periods (Spinillo & Bryant, 1999; Van Dooren et al., 2010). On the other hand, this skill develops slowly and some students cannot acquire it even in later academic years (Hoffer, 1998). However, the National Council of Teachers of Mathematics (1989) calls attention to the significance of proportional reasoning skills and claims that all requirements should be met for developing proportional reasoning skills regardless of the amount of time and effort they require.

Upon examining the literature (Akkuş & Duatepe-Paksu, 2006; Cramer et al., 1993; Heller et al., 1990; Pişkin-Tunç, 2020; Post et al., 1988), one can see that the evaluation of proportional reasoning skills occurs through various problems types. These are missing value problems, quantitative comparison problems, and qualitative reasoning problems. Additionally, researchers use problems that involve non-proportional type relationships to determine whether there are proportional and non-proportional conditions. A missing value problem concerns the discovery of the fourth value while three of the multiples are present in an a/b=c/d proportion (Lamon, 2007). A typical example of one of the most common missing value problems one can encounter in school math is as follows: "How many hours would it take for a car that goes 300 kilometers in 4 hours to go 750 kilometers with the same speed?" (Akkuş & Duatepe-Paksu, 2006, s.9) However, numerical comparison problems are problems that present the values a, b, c, and d in a proportion indicating the equality of two ratios such as a/b and c/d and require the a/b and c/d ratios to be compared (Lamon, 2007). The orange juice problem (Noelting, 1980) is an example of this type of problem. This problem requires students to determine which of the orange juice mixtures created in two separate jugs by mixing two different ratios of orange juice concentrates and water is sweeter. On the other hand, qualitative reasoning problems are problems that do not present numerical values and therefore one must make a comparison without relying on numerical values. The following is an example of this type of problem: "On a running track, Elif ran more laps in less time than Emel. Who is the faster runner? Write the explanation" (Akkuş & Duatepe-Paksu, 2006, p. 10).

Proportional Reasoning Strategies

Strategies used to solve proportion problems are generally classified as formal and informal strategies (Baroody & Coslick, 1998; Hood & West, 1994). In this classification, algebraic strategies (cross-multiplication) using algebra rules are classified as formal strategies, and strategies using proportional relationships (unit ratio, factor of change, etc.) are classified as informal strategies. Cramer and the Post (1993) state that one should emphasize informal strategies in solving proportion problems and should not teach formal strategies until students fully internalize these strategies. However, studies (Ayan &Işıksal-Bostan, 2019; Ben-Chaim et al., 2012; Cramer & Post, 1993; Özgün-Koca & Kayhan-Altay, 2009; Toluk-Ucar & Bozkuş, 2018) show that students use the cross-multiplication strategy, which is more of a memorization procedure, in the solution of proportion problems. In this strategy, it is possible to establish proportions with the cross-multiplication algorithm and solve the equation (Van de Walle, Karp, & Bay-Williams, 2010). On the other hand, informal strategies used in the solution of proportion problems are the building-up, factor of change, unit rate, and equivalent fraction strategies. The building-up strategy is based on the use of an additive pattern to achieve the desired multitude (Lamon, 2007). For example, an example solution using the building-up strategy to the problem of "How many kilometers does a vehicle that travels 200 kilometers in 2 hours at a fixed speed travel in 6 hours?" can be as follows: If the vehicle travels 200 km in 2 hours, it travels 400 km (200 km+200 km) in 2 more hours (4 hours in total), and 600 km (400 km+200 km) in another 2 hours (6 hours in total). When using the building-up strategy, people consider it as an intuitive strategy in which one does not take into account the multiplication relationship between quantities and people do not accept it as a process in which proportional reasoning is used without additional knowledge (Lamon, 2012). Another informal strategy is the factor of change strategy in which one finds the multiplicative relationship between multiples (Cramer et al., 1993) with the question of "How many times?" (Cramer & Post, 1993). Let us explain how to use this strategy based on the example above. In the process of solving how many km a vehicle that travels 200 km in 2 hours at a constant speed (first case) can travel in 6 hours (second case) using a factor of change strategy, one should derive that if time has tripled, the road length will also triple. In other words, in this process, first of all, one calculates how much more time is spent traveling, then one multiplies the road length in the first state with this factor of change and they calculate the road length in the second state. Another informal strategy is the unit rate strategy to find multiplicative relationships between multiples through division (Cramer et al., 1993) with the question of "How many for one?" (Cramer & Post, 1993). Let us explain how to use this strategy based on the example above. In the process of solving how many km a vehicle travels in 6 hours if it travels 200 km at a constant speed in 2 hours, one calculates how many km the vehicle travels in 1 hour using the unit ratio strategy. Then, one multiplies the travel time (6 hours) by the unit road length and reaches the desired result. The equivalent fractions strategy involves perceiving the rates as equivalent fractions and creating fractions that are equivalent to the given fraction (Duatepe et al., 2005). On the other hand, in addition to these strategies that lead to the right solution upon appropriate usage, there are strategies that lead to the wrong solution. The most common one is the additive relationship strategy of using additive relationships instead of multiplicative relationships (Ben-Chim et al., 2012). In the process of using this strategy, also called the incorrect additive strategy, there is an attempt to solve the proportional questions by subtracting certain numbers from the multiples that make up a rate or by adding certain numbers to the multiples (Lamon 2007; Lamon, 2012).

Rationale

Although there is a great emphasis on proportional reasoning, many studies have found that students experience difficulties (Lobato & Thanhesier, 2002; Modestou & Gagatsis, 2007). According to Lesh et al.

(1988), this situation is the result of limiting the skill of proportional reasoning to solving missing value problems, in turn this causes information to stay at a surface level and be limited. However, one cannot consider solving missing value problems equal to proportional reasoning (Cramer & Post, 1993; Lesh et al., 1988). This is because proportional reasoning requires the ability to solve different proportional problems (Karplus et al. 1983; Cramer & Post, 1993), distinguishing proportional relationships from non-proportional relationships, and understanding the mathematical relationships within proportional situations. In other respects, for researchers, students' considering non-proportional relationships as proportional ones, and therefore using proportional strategies while solving non-proportional problems is one of the most important problems (Degrande et al., 2017; Modestou & Gagatsis, 2007; Van Dooren et al, 2007). Research explains these incorrect tendencies with students' transition from additive reasoning to multiplicative reasoning (Siemon, Breed & Virgona, 2005), the superficial inclusion of reasoning in schools (De Bock et al., 1998; Van Dooren et al., 2008), and the features of mathematical tasks (Degrande et al., 2017; Van Dooren et al., 2010).

Studies on proportional reasoning (Aladağ & Artut, 2012; Arıcan, 2019; Ayan & Işıksal-Bostan, 2018; Ayan & Işıksal-Bostan, 2019; Avcu & Doğan, 2014; Çelik & Özdemir, 2011; Çomruk, 2018; Kahraman et al., 2019; Mersin, 2018; Öztürk et al., 2021; Pakmak, 2014; Yılmaz-Özen, 2019) focus specifically on examining the proportional reasoning skills of middle school students or identifying strategies used in problems involving proportional situations. Also, there are a small number of studies (Atabaş, 2014; Pelen & Dinç-Artut, 2019; Pişkin-Tunç, 2020; Toluk-Uçar & Bozkuş, 2016) that were carried out with students at the elementary and/or middle school level that involve differentiating between proportional and non-proportional problems. However, proportional reasoning is an important skill not only at the middle school level but also at later levels. Therefore, what strategies both middle school level and high school level students use when solving proportional and non-proportional problems and whether these strategies differ are subjects of interest. To fill in this gap within the literature, this study aimed to examine the strategies students between 7th and 12th-grade used to solve proportional and non-proportional problems. The examination of the strategies students use provides us with significant information about their ability to differentiate proportional relationships from non-proportional relationships and therefore their proportional reasoning. In this context, the research seeks to answer the question "What strategies do students at different grade levels (7th-12th grade) use to solve proportional and non-proportional problems?"

METHOD

The Research Design

This study used a cross-sectional research pattern to determine the development of proportional reasoning skills of students at different grade levels (7th-12th grade). Cross-sectional research is a descriptive survey model (Creswell, 2012) used to collect data from individuals of different age groups at a given time. Due to the difficulty of keeping track of the same students for a long time, researchers collected data from students at different grade levels at the same time, and determined and compared the strategies that students used in proportional and non-proportional problems.

The Study Group

The study group of this research includes 49 middle school (7th-8th grade) and 130 high school (9th-12th grade) students. As seen in Table 1 the study group consists of 24 seventh graders (12-13 years old) and 25 eighth-graders (13-14 years old) from a middle school, 38 ninth-graders (14-15 years old), 33 tenth-graders (15-16 years old), 32 eleventh-graders (16-17 years old) and 27 twelfth-graders (17-18 years old) from a high school.

Table 1. The Study Group

	Middle School		**High School**			
Grades	**7th grade**	**8th grade**	**9th grade**	**10th grade**	**11th grade**	**12th grade**
n	24	25	38	33	32	27
Age	12-13 years	13-14 years	14-15 years	15-16 years	16-17 years	17-18 years

In the process of forming the study group, the researcher chose two schools in the same neighborhood with similar mathematics achievement level, one middle school and the one high school, and randomly selected the classes from each grade level to perform the study. The schools were located in a neighborhood where families with middle socioeconomic income level live. It was tried to ensure continuity in the high school student profile as a continuation of the middle school by selecting the study group in schools in the same neighborhood. Thus, the study group was formed in accordance with the cross-sectional research design.

Data Collection Tool

In the process of creating the data collection tool, first, the researcher prepared a 12-question draft measuring tool consisting of four questions from the first three problem groups based on the literature (Akkuş & Duatepe-Paksu, 2006; Atabaş, 2014; Cramer et al., 1993; Hillen, 2005; Karplus et al., 1983; Mudestou & Gagatsis, 2007; Pişkin-Tunç, 2020). The researcher consulted the opinions of 4 teachers, two middle school and two high school teachers, to check whether this measurement tool was suitable for the students' level and whether it was clear. Accordingly, some changes were made to the draft measuring tool. Then, two researchers who studied proportional reasoning reviewed the measurement tool. At the suggestion of one of them, the researcher decided to add a non-proportional problems group to the data collection tool. Once again, experts gave their opinions on the new data collection tool formed with the addition of this problem group, and the data collection tool was finalized in line with the feedback received. Thus, the data collection tool consists of the missing value problems Problem 1 (P1) and Problem 6 (P6), the numerical comparison problems Problem 3 (P3) and Problem 4 (P4), the qualitative reasoning problems Problem 5 (P5) and Problem 8 (P8), and the non-proportional problems Problem 2 (P2) and Problem (P7). Therefore, data collection occurred through a data collection tool consisting of a total of 8 problems in 4 different groups. Table 2 presents the problems used in the study.

Table 2. The Problems in the Data Collection Tool

Problem type	Problem
Missing value problems	**P1)** Ali and Ayşe want to enlarge their friends' photos for the graduation yearbook without ruining them. One photo they want to enlarge is 4 cm in length and 3 cm in width. If this photo is 14 cm in length after being enlarged, how many cm is its width? **P6)** Emre and Sıla buy books that are on sale from the bookstore. The books on sale have the same price. Emre buys 3 books and Sıla buys 8 books. Since Emre pays 15 TL, how much will Sıla pay for the books she bought?
Numerical comparison problems	**P3)** Vehicle A traveled 180 km in 3 hours and vehicle B traveled 400 km in 7 hours. Which vehicle was driven faster? Write an explanation. **P4)** When two friends went to the market, they saw that 2 liters of orange soda was 6 TL and 6 liters of lemonade was 15 TL. They decided to buy orange soda. Do you think they made an economical choice? Why?
Qualitative reasoning problems	**P5)** A mother makes juice for her daughter every day by mixing apples and oranges. If the mother used fewer oranges and fewer apples than yesterday, then the taste of the juice would: (a)Taste more like oranges than yesterday, (b) Taste more like apples than yesterday, (c) Be the same as yesterday, (d) The information provided is not enough. **Explanation:** **P8)** Umut ran fewer laps today than he did yesterday in more time. Accordingly, Umut's running today compared to yesterday is; a) faster b) slower c) the same d) the information provided is not enough. **Explanation:**
Non-proportional problems	**P2)** On a sunny day, two T-shirts dry in 30 minutes. According to this information, how many minutes does it take for 4 T-shirts to dry in the same weather conditions? **P7)** Ali and Ahmet run at equal speeds on a running track. Ali started running first. When Ali has ran 9 laps, Ahmet has ran 3 laps. How many laps does Ali run when Ahmet completes 15 laps?

There were sufficient space for the solution under each problem in the data collection tool and the participants were asked to write their solutions in these spaces. Participants had one forty minutes to solve the problems. The ethics committee approval of this study was obtained from the Social and Human Sciences Publication Ethics Committee of Ordu University with the decision numbered 2022-99.

Data Analysis

The study used the descriptive analysis method to analyze data. This process consists of analyzing the data according to the previously determined themes. In the literature, various strategies used in proportional reasoning problems are the unit rate, factor of change, cross-multiplication algorithm, equivalent fraction, equivalence class, emotional strategies, etc. When examining the solutions of students for missing value (P1, P6) and numerical comparison problems (P3, P4), the researcher used the codes of "cross-multiplication", "factor of change", "unit rate", "equivalent fractions", "building-up", and "inaccurate additive strategy". Solutions that did not comply with these previously determined strategies were coded as "not clear" and those that did not have any solutions were coded as "no solution". On the other hand, the researcher analyzed the strategies used to solve qualitative problems (P5, P8) and non-proportional problems (P2, P7) in line with the framework used by Pişkin-Tunç (2020). The solutions of the qualitative problems were first classified as "including multiplicative comparison" and "not including multiplicative comparison". In this process, for the solutions that include multiplicative comparison, those of which presented numerical examples with a quantitative comparison were coded as "quantitative multiplicative comparison" and comparisons without numerical values were coded as “qualitative multiplicative comparison”. Morever the researcher used the codes of "non-proportional strategy" and "incorrect proportional strategy" for the analysis of solutions to non-proportional problems. For the reliability of coding, another researcher coded the answers of 10 students randomly selected from each grade level and the percentage of agreement between the coders found to be 0.96, 0.93, 0.91, and 0.87, respectively, according to the problem types presented in Table 2 based on the formula proposed by Miles and Huberman (1994). The study presented the percentages of strategies used for each problem with tables or charts, thus allowing one to make comparisons in and between grade levels. Additionally, the study exemplified the strategies used through direct excerpts from student solutions.

FINDINGS

This section presents the findings obtained within the scope of the research based on the types of problems. Considering the research design in the presentation of the findings facilitated the comparison of solution strategies according to grade levels.

Strategies Used in Missing Value Problems

Table 3 presents the percentages of strategies that students of different level used to solve missing value problems (P1 and P6).

Table 3. Percentages of Strategies Used in Missing Value Problems (P1, P6) by Grade Levels

	STRATEGIES	Middle School		High School			
		7th grade	8th grade	9th grade	10th grade	11th grade	12th grade
P1	Cross-multiplication	16.7	20	21.1	18.2	28.1	11.1
	Factor of change	4.2	4	7.9	9.1	12.5	11.1
	Unit rate	0	0	0	0	0	0
	Building-up	4.2	0	0	0	0	0
	Equivalent fractions	0	8	0	0	12.5	18.5
	Inaccurate additive	41.7	52	42.1	48.5	34.4	44.4
	Not clear	12.5	12	28.9	24.2	9.4	11.1
	No solution	20.8	4	0	.0	3.1	3.7
P6	Cross-multiplication	70.8	72	71.1	63.6	62.5	63
	Factor of change	4.2	4	7.9	27.3	28.1	14.8
	Unit rate	0	4	0	0	0	11.1
	Building-up	0	0	2.6	3	3.1	0
	Equivalent fractions	0	0	0	0	0	0
	Inaccurate additive	0	0	0	0	0	0
	Not clear	8.3	16	10.5	3	3.1	3.7
	No solution	16.7	4	7.9	3	3.1	7.4

In P1, findings indicate that the majority of students at all grade levels solved the question using the inaccurate additive strategy. It was observed that 41.7% of 7th graders, 52% of 8th graders, 42.1% of 9th graders, 48.5% of 10th graders, 34.4% of 11th graders, and 44.4% of the 12th graders think additively by focusing on the difference between the edges of the two rectangles as in Figure 1.

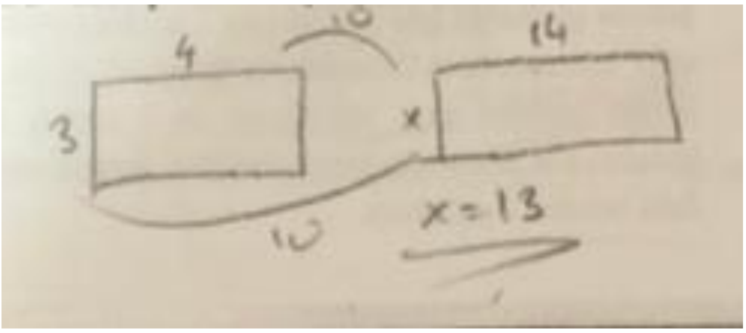

Figure 1. The Solution of a 10th Grader that Used the Inaccurate Additive Strategy in P1

On the other hand, one can derive that in P1, students solved the questions by resorting to 4 different strategies of appropriate reasoning (cross-multiplication, factor of change, building-up, and equivalent fractions). Upon examining these solutions, one can understand that the percentage of students using the cross-multiplication strategy at each grade level is higher than the percentage of students who use other strategies. The following is the solution of a 7th grade student who uses the cross-multiplication strategy in the solution of P1:

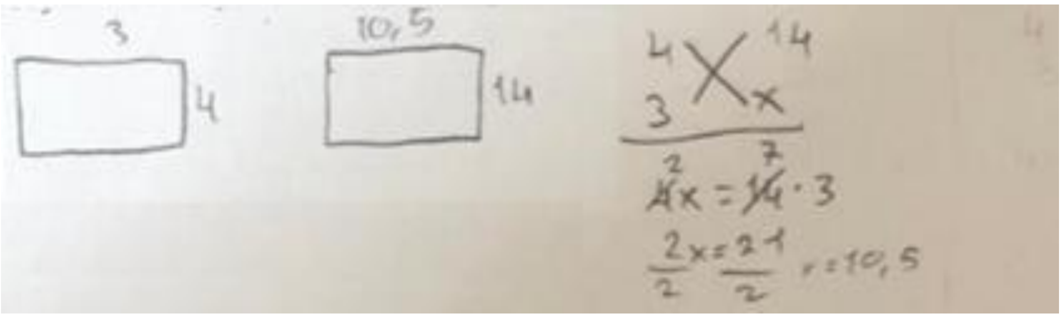

Figure 2. The Solution of a 7th grader that Used the Cross-multiplication Strategy in P1

On the other hand, findings indicated that the majority of all grade level students use the cross-multiplication strategy in P6. It was observed that 70.8% of 7th graders, 72% of 8th graders, 71.1% of 9th graders, 63.6% of 10th graders, 62.5% of 11th graders, and 63% of the 12th graders were as so. Additionally, in Table 3 one can observe that 10th, 11th and 12th grade students use the factor of change strategy more in their solutions than other grade-level students, and also that only 8th and 12th grade students used the unit rate strategy. It is also noteworthy that no students used the inaccurate additive strategy based on inaccurate reasoning for P6, which is a typical missing value problem.

Strategies Used in Numerical Comparison Problems

Table 4 presents the percentages of strategies that students at different levels used to solve numerical comparison problems (P3 and P4).

Table 4. Percentages of Strategies Used in Numerical Comparison Problems (P3, P4) by Grade Levels

	STRATEGIES	Middle School		High School			
		7th grade	**8th grade**	**9th grade**	**10th grade**	**11th grade**	**12th grade**
P3	Cross-multiplication	0	4	0	3	3.1	0
	Factor of change	8.3	12	18.4	15,2	28.1	22.2
	Unit rate	62.5	64	63.2	72.7	65.6	74.1
	Building-up	0	12	2.6	3	0	0
	Not clear	16.7	8	7.9	3	3.1	3.7
	No solution	12.5	0	7.9	3	0	0
P4	Cross-multiplication	0	0	0	0	0	0
	Factor of change	37.5	40	26.3	27.3	34.4	33.3
	Unit rate	33,3	36	47,4	48.5	53.1	63
	Building-up	0	8	2.6	3	0	0
	Not clear	25	16	21,1	21.2	12.5	3.7
	No solution	4.2	0	2.6	0	0	1.1

Notably, the most commonly used strategy at each grade level in P3 is the unit rate strategy. It was observed that 62.5% of 7th graders, 64% of 8th graders, 63.2% of 9th graders, 72.7% of 10th graders, 65.6% of 11th graders, and 74.1% of 12th graders used the unit rate strategy in their solutions. One can derive that especially 11th and 12th grade high school students use the factor of change strategy more than students at other levels. In P4, however, one can observe that the factor of change and unit rate strategies appear prominent at all class levels. Base-d on a comparison between grade levels, 7th-8th grade students use the factor of change strategy more, and 9th-12th grade students use the unit ratio strategy more. Also, as the grade level increases in P4, the percentage of students who use the unit rate strategy increases. The following is a 10th grade students' response example to P3, in which the student used the unit rate strategy and stated that "A 60 per hour, B 57,… per hour. A was driven faster" in his written explanation.

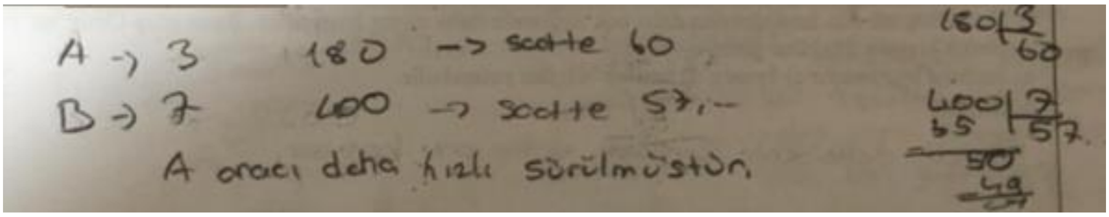

Figure 3. The Solution of a 10th Grader that Used the Unit Rate Strategy in P3.

Qualitative Reasoning Problems

Figure 4 presents the distribution of strategies that the students used to solve the qualitative reasoning problems of P5 and P8. This figure indicates that the majority of students in 9th-12th grade can perform qualitative multiplicative comparison in both problems.

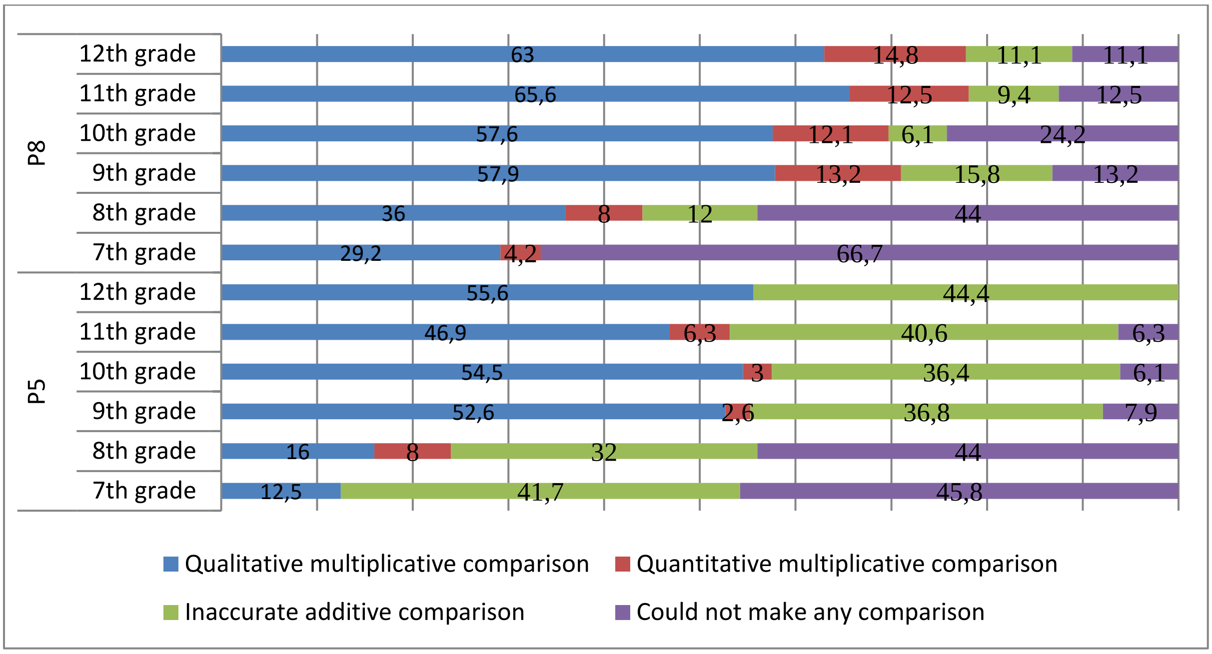

Figure 4. Percentages of Strategies Used in Qualitative Reasoning Problems (P5, P8) by grade levels.

The following is a 12th grade student's answer to P8 in which they used the qualitative multiplicative strategy. In his written explanation the student stated that "He ran yesterday. Less laps in more time today... He is slow because given more time, he still ran less".

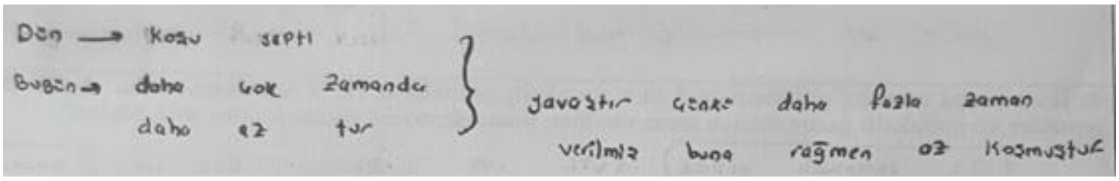

Figure 5. The Solution of a 12th Grader that Used the Qualitative Multiplicative Comparison Strategy in P8.

Additionally, to make comparisons quantitative values were determined and answers that used the quantitative multiplicative comparison strategy were found with a lesser percentage at each grade level. In Figure 6 the student states that "They ran 1 km in 1 hour yesterday and 500m in 1.5 hours today. Today, he has run less in more time than yesterday".

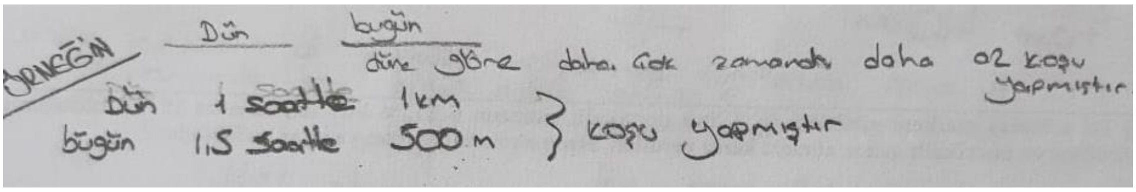

Figure 6. The Solution of a 9th Grader that Used the Quantitative Multiplicative Comparison Strategy in P8.

According to the findings the number of students who did inaccurate additive comparison is greater in P5 than P8 at each grade level. It was observed that 41.7% of 7th graders, 32% of 8th graders, 36.8% of 9th graders, 36.4% of 10th graders, 40.6% of 11th graders, and 44.4% of 12th graders answered this question in a way that fits the category of inaccurate additive comparison by using additive thinking. The answers of these students were often "the same as yesterday", and one can see that they did not mention the ratio of apples and oranges as in explanations such as "because if both reduce equally, there will be no change in the taste of the juice, only the amount will decrease". Some students elaborated on the decreasing amount of apples and oranges by selecting "the information provided is not enough" and making statements such as "because there is no information regarding the amount the oranges or apples were reduced compared to yesterday".It is noteworthy that the percentage of students, whose answers were classified in the "could not make any comparison" category, as they did not make any explanations for both questions, is quite high, especially at the 7th and 8th grade levels. One can observe that for P5 45.8% of 7th graders and 44% of 8th graders; and for P8, 66.7% of 7th graders and 44% of 8th graders simply selected one of the options and/or did not make any statements.

Strategies Used in Non-Proportional Problems

Figure 7 presents the distribution of the strategies that students used in solving the non-proportional problems of P2 and P7.

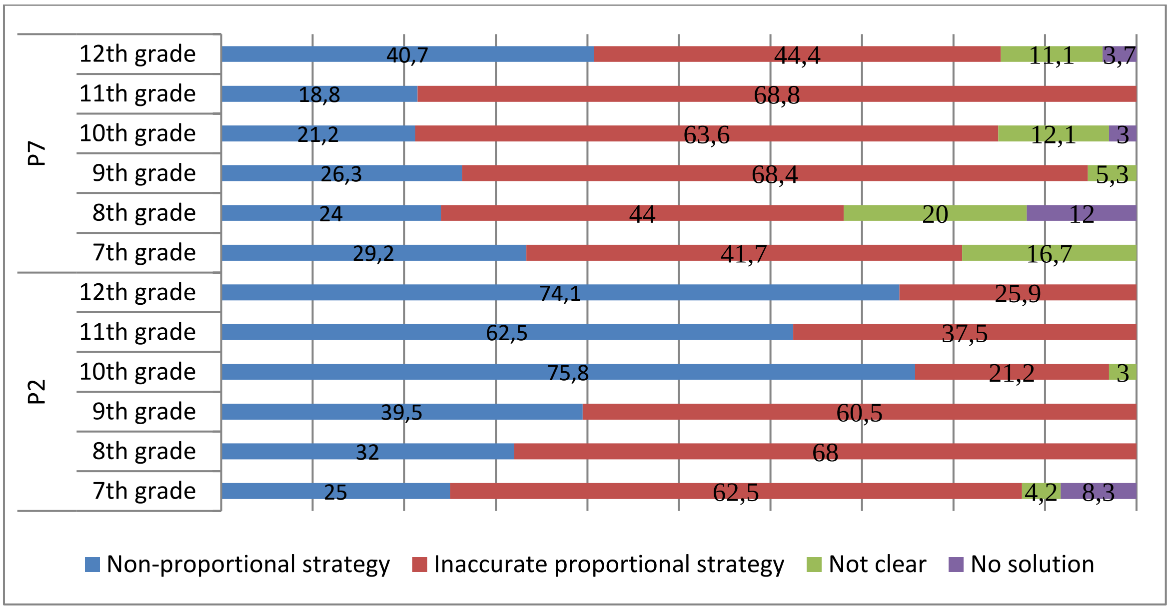

Figure 7. Percentages of Strategies Used in Non-proportional Problems (P2, P7) by Grade Levels.

Upon examining Figure 7, one can understand that the majority of 7th, 8th, and 9th graders used the inaccurate proportional strategy, and the majority of 10th, 11th and 12th graders solved the problem using the non-proportional strategy in P2. Notably, the majority of students at each grade level solved the problem using the inaccurate proportional strategy for P7, as seen in Figure 8. The student stated that “Direct proportion because they run at equal speeds, x=45 laps”.

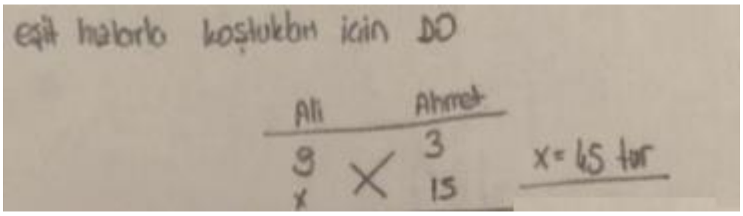

Figure 8. The Solution of a 8th Grader that Used the Inaccurate Proportional Strategy in P7.

Additionally, for this question, one can observe that 12th grade students had a higher percentage (40.7%) than other grade level students in using a non-proportional strategy based on additive reasoning, as in Figure 9. In his written explanation the student stated that “If the speeds are equal, the number of laps will also be equal. That is, the difference between them does not change. There are 6 rounds difference. If Ahmet did 15 laps in the first place, then Ali will do 21 laps”.

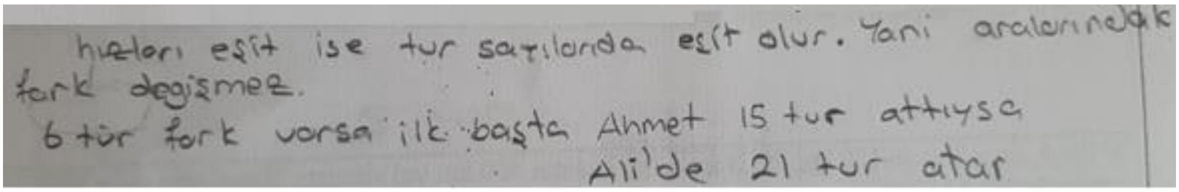

Figure 9. The Solution of a 9th Grader that Used the Non-proportional Strategy in P7.

CONCLUSION and DISCUSSION

This study aimed to examine the strategies that students from different grade levels (7th-12th grade) used in proportional and non-proportional problems. The findings from the study indicate that the strategies students mostly used in solving two different missing value problems in the data collection tool were different. Cross-multiplication was the most used strategy at every grade level for P6, which is a typical missing value problem that can be found in almost every textbook on the subject. Many studies (Ayan &Işıksal-Bostan, 2019; Ben-Chaim et al., 2012; Cramer & Post, 1993; Özgün-Koca & Kayhan-Altay, 2009; Pişkin-Tunç, 2020; Toluk-Uçar & Bozkuş, 2018) also indicate that students mostly use the cross-multiplication strategy, a formal strategy that does not emphasize multiplicative relationships (Lamon, 2007) when solving proportional problems. However, in P1, which is also a missing value problem, the majority of all students solved the problem using the inaccurate additive strategy. The variables in this problem include non-integer ratios. These different results in the two different proportional reasoning problems from the same group suggest that the context of the problem has an impact on this situation. In fact, studies (Cramer & Post, 1993; Degrande et al., 2017; Fernández et al., 2010; Heller et al., 1989; Hood & West, 1994; Van Dooren et al., 2010) address the challenges students face in proportional reasoning due to the effects of the context of the problem on students' strategy choices and their additive or multiplicative reasoning preferences. In her study, Pişkin-Tunç (2020) stated that 6th graders can think proportionally in the context of speed, which is a common context for proportional problems, but in the contexts of mixture and scaling problems, which are not as common; students often use additive strategies that are wrong. Other studies (Cramer et al., 1993; Karplus et al., 1983; Pişkin-Tunç, 2020;

Singh, 2000) indicate that when proportional problems involve non-integer ratios, students often use inaccurate additive strategies. The researcher believes that the students' use of this strategy based on inappropriate reasoning in the solution of P1 is due to these reasons.

Unlike the missing value problems, findings determined that students generally solved one of the numerical comparison problems (P3) using the unit rate strategy, regardless of grade level. Also, the study found that 7th and 8th graders used factor of change strategies more in another numerical comparison problem (P4). In the literature, it is stated that students use these two strategies more in numerical comparison problems. For example, Pelen (2014) states that 6th graders use the factor of change strategy the most; Duatepe et al. (2005) state that 6th, 7th, and 8th-grade students use the unit ratio strategy the most, and Kahraman et al. (2019) state that 7th-grade students solve problems using the unit ratio strategy the most. This study found no notable difference with regard to grade level for the type of strategies used in numerical comparison problems.

The study also found that the majority of high school students could make qualitative multiplicative comparisons in qualitative reasoning problems, while the majority of middle school students could not make any comparisons. The reason that middle school students experienced more difficulties than high school students may be due to the fact that their learning background relating to ratio subjects was shorter than that of high school students. Although problems of this nature are not presented by teachers and textbooks (Pişkin-Tunç, 2016), it is possible that the proportional reasoning of high school students may have been supported while learning other subjects related to proportional concepts during students’ ongoing education. On the other hand, the findings determined that approximately 40% of students at each grade level gave answers based on additive reasoning in P5. It is noteworthy that there is a difference between the percentages in this category although they are in the same problem group. The researcher believes this is because P5 is more complex for students than P8.

The study found that 7th, 8th and 9th graders used an inaccurate proportional strategy in the first of the non-proportional problems (P2). This finding suggests that these students did not notice the constant relationship between the variables and were unable to distinguish between proportional and non-proportional conditions. The majority of 10th, 11th, and 12th graders on the other hand were able to determine that the relationship between variables was constant in P2. The students who solved this non-proportional problem with a constant relationship with the right strategy were students from more advanced grade levels. On the other hand, in the other non-proportional problem (P7), the majority of students in all grade levels did not notice the additive relationship between the variables, so they answered the question with multiplicative thinking and used an inaccurate proportional strategy. This finding suggests that these students have difficulty distinguishing between proportional and non-proportional situations. Modestou and Gagatsis (2009), who reached similar results, stated that students at certain grade levels directly use similar strategies learned without looking at the context of the problem as a result of the effect of education. Within the scope of this study, it is thought that the students' use of multiplicative strategies in non-proportional problems stems from a similar reason. This situation, which refers to the use of proportional reasoning in cases where there are non-proportional relationships and in which proportionality is seen as an overgeneralization according to the literature, is found in many studies in the literature (Atabaş, 2014; Pişkin-Tunç, 2016; Pişkin-Tunç, 2020; VanDooren et al., 2005).

On the other hand, the research findings show that the type of problem is effective in the strategies the students use in the solutions. In missing value problems, the most commonly adopted strategy was cross-multiplication when the problem was a typical missing value problem encountered at every grade level and inaccurate additive strategy in cases in which the problem was less familiar. In one of the numerical comparison problems, the most used strategy at each grade level was unit rate; while in the other problem, the most used strategies were factor of change and unit rate. However, the study could not find a clear trend regarding which strategies students used in missing value and numerical comparison problems as the grade level increased. Further, the study determined that high school students were able to make much better multiplicative comparisons than middle school students in qualitative reasoning problems. However, there were no specific trends due to grade level in relation to the percentage of students who did inaccurate additive comparison in these problems. For the non-proportional problems, the study determined that the majority of 7th, 8th, and 9th grade students in the problem containing a constant relationship (P2), and students of all class levels in the problem that included an additive relationship (P7) made incorrect reasoning by thinking multiplicatively. Similarly, there are studies in the literature that indicate that students at different grade levels are more successful in different types of problems. For example, Atabaş (2014) states that 5th and 6th-grade students showed the highest success in the missing value problem and the lowest success in the constant relationship problem from the problems in the four different groups of missing value, numerical comparison, additive, and constant problems.

Focusing solely on finding the value that is not given in a proportion leads to implementing the rules without thought and therefore the proportional reasoning skills of the students do not develop (Van de Walle, Karp, & Bay-Williams, 2010). Researchers view the emphasis that traditional proportion teaching puts on the memorization of rules and memorizing calculations a problem related to learning and teaching proportional relationships (Izsák &Jacobson, 2013; Singh, 2000). The findings from this study indicate that students adopted a cross-multiplication strategy in typical proportion problems they encountered before. In cases in which they were not familiar with the problem, instead of making memorized calculations, students resorted to various ways of reasoning with the use of different strategies.

RECOMMENDATIONS

Upon examining the findings generally, the study found that the type, context, and numerical structure of the problem are effective in the strategy choices of the students. From this viewpoint, it is necessary to enrich the problems used in mathematics courses and in the textbooks, which are the primary sources. Not only missing value problems, but also numerical comparison problems and qualitative reasoning problems should be included in mathematics textbooks and mathematics courses. The solutions of the problems included in the lessons and textbooks should not be limited to the cross multiplication strategy, and efforts should be made to support the proportional reasoning skills of the students. Teachers should encourage their students to use informal strategies when solving proportionality problems. In this process, the solution phase should not be started immediately by applying formal strategies, appropriate guidance should be given when necessary for the use of informal strategies by enabling students to think more about the problem situation. Moreover, classroom discussions should be conducted on the tasks that will enable to distinguish proportional relationships from non-proportional relationships. This study brought forth the strategies that students used by examining their written solutions. It is recommended that future studies conduct in-depth examinations about which strategies students choose and how they use them through clinical interviews.

ETHICAL TEXT

In this article, journal writing rules, publication principles, research and publication ethics rules, journal ethics rules were followed. Responsibility for any violations that may arise regarding the article belongs to the author. In addition to these, ethical approval was obtained from the Ordu University Social and Human Sciences Ethics Committee with the decision numbered 2022-99 for the implementation of the current study.

Author(s) Contribution Rate: The author's contribution rate in this study is 100%.

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