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INVESTIGATION OF MATHEMATICAL MODELLING PROCESS IN ARGUMENTATION BASED DISCUSSIONS: THE CASE OF SSI[1]

Mustafa Çağrı GÜRBÜZ

Assistant Professor, Istanbul Aydin University, Faculty of Education, Istanbul, Turkey, mcggurbuz@gmail.com

ORCID: 0000-0003-1851-2672

Şirin YILMAZ

Assistant Professor, Istanbul Aydin University, Faculty of Education, Istanbul, Turkey, sirinyilmaz87@gmail.com

ORCID: 0000-0002-0238-8550

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ABSTRACT

Socio-Scientific Issues (SSI) can aid the development of logical decision-making skills in students, thereby helping schools develop knowledgeable individuals. When students engage in modelling tasks during the transition from school mathematics to the real world, their cognitive reasoning can be supported in a practical context during model development. Since it is speculated that the arguments developed in the modelling cycle are derived from modelling procedures, exploring the modelling procedures may offer insights into the students' arguments. The purpose of this study is to analyse how pre-service teachers deploy mathematical models while discussing the context of COVID-19, a socio-scientific issue, and to examine how the modelling process is linked. This study aims to investigate how pre-service teachers engage with argumentation and modelling practices and how one practice may impact the other. Our goal is to explore how the argumentation process concerning the phenomenon is interconnected with the modelling process. To achieve this, we have designed a learning environment that facilitates teacher candidates' participation in scientific argumentation and modelling practices. The research findings were analysed by investigating the sub-arguments formed during the modelling cycle and considering how the modelling processes corresponded with the research question. Upon close examination of the argument structures utilised by aspiring teachers in their solution proposals and modelling, it became evident that the majority of structures were of a lower level, with few high-level models.

Keywords: Mathematical modelling, socio-scientific issues, argumentation, teacher candidates.

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INTRODUCTION

Today, more value is given to methods that make individuals active in learning-teaching processes, encourage them to discuss and take the subject with different aspects. Modelling and argumentation can be given as examples of these methods. One of the main purposes of science is to elaborate explanations for the data collected when the questions proposed by researchers are investigated. Scientific explanations relate to the reasons why something occurs. Scientific explanations tend to link causes and effects and establish relationships based on evidence and logical arguments (Berland & Reiser, 2009; Braaten & Windschitl, 2011; National Research Council, 2012).

Scientific discussion used in teaching processes is basically a process that takes place in the form of mutual dialogues between two or more people. If a dialogue guides the coordination of theory and evidence in putting forward an explanation, example, prediction or evaluation about a relevant topic, such a special situation can be defined as scientific discussion (Duschl & Osborne, 2002). Debate is a focal point for educational endeavours, as well as an approach that makes irregular applications of science and enables reasoning about claims, discussing evidence or counterarguments, and the acceptance of data and theories in society (Sadler & Fowler, 2006).

In recent years, the science and mathematics education framework has been concerned with making the classroom environment more intellectually engaging in order to do more than present science as a static body of knowledge through didactic teaching practices. In order to achieve this aim, firstly, the focus is on how individuals learn, and classroom environments are created that involve the student as an active participant in understanding and making sense of scientific ideas, which is the goal of teaching (Bransford, Brown, & Cocking, 2003). Secondly, students are expected to do more than just learn knowledge; the focus is on students' understanding of the scientific enterprise by participating in activities similar to what scientists actually do. The production of skills from knowledge and the creation of a product are emphasised. Learning scientific content and engaging in authentic scientific practice are not mutually exclusive. One potential way to realise these two aims could be to create opportunities for students to engage in scientific argumentation in the classroom. Argumentation has the potential to support both content learning and authentic learning experiences. Engaging in argumentation in authentic ways is known to have the potential to support learning in a variety of dimensions, from content to process to epistemology (Svoboda & Passmore, 2011).

Considering the uncertainty of scientific knowledge, the process of generating, testing, evaluating and revising models is called modelling. Modelling processes can be seen as central to the development, critical evaluation and dissemination of scientific knowledge (Nersessian, 2002). The model-based view of science offers researchers the opportunity to make choices and justify them; to propose intermediate models; to communicate them to their peers; to plan and investigate to collect data to evaluate their proposals; to criticise their models and those of their peers; and to modify models according to specific criteria or evidence. In this sense, the modelling process is inherently critical and argumentative (Boettcher & Meisert, 2011; Giere, 2001; Svoboda & Passmore, 2011; Windschitl, Thompson, & Braaten, 2008).

Argumentation and modelling can be considered as fundamental scientific practices in learning processes. Research in these areas shows that the inclusion of certain practices in mathematics and science teaching can attract students' interest (Evagorou, Nicolaou & Lymbouridou, 2020; Fischer, et. al., 2018; Eurydice, 2012). Other studies (Hodson, 1992; Millar & Osborne, 1998; National Research Council, 2012; Teaching and Learning Research Programme, 2006) have emphasised that students' participation in public debates involving scientific components should be a common aspect of education to support the development of their scientific reasoning.

Since the early 1990s, mathematical modelling has started to be included in curricula, especially in the United States and European countries. Similarly, one of the main objectives of mathematics education in many countries is to include mathematical models and applications in their curricula (Kaiser, 2015). In recent years, mathematical modelling has become increasingly important (Dede, 2019; Kaiser & Schwarz, 2006). It is not easy to describe a modelling conception and its epistemological background (Kaiser & Sriraman, 2006). The mathematical modelling used in this study is a cyclical process that requires the mathematical expression of a real-life problem, the solution of the mathematical problem, and the interpretation of the results obtained from the solution in the context of mathematics (Blum & Niss, 1991). Figure 1 shows the mathematical modelling cycle created by Blum and Leiß (2005).

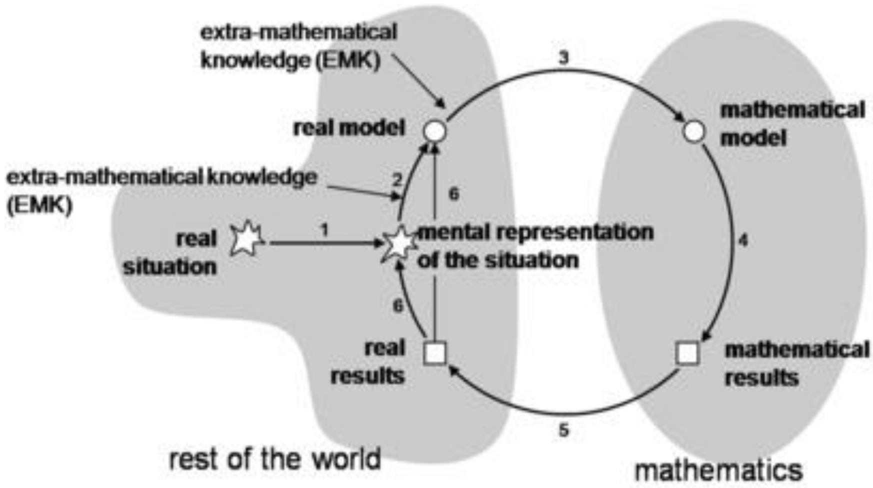


Figure 1: Mathematical Modelling Cycle (Blum & Leiß, 2005)

To explain the model cycle that is tried to be expressed in Figure 1; 1 It begins with understanding the task. 2. It is the simplification or construction of the task. 3. Mathematization is the part where the extra mathematical information from the problem situation is intense. 4. It is necessary to work with mathematics, personal mathematical competences are used in this section. 5. Interpreting. 6. Validating. Blum and Leiß (2005) state that in the modelling cycle, an individual first makes sense of the problem when faced with a real-life problem and makes a mental model of the problem. The individual transforms this mental model created for the problem situation into a real model by distinguishing and simplifying the necessary and unnecessary variables. It transforms this model into a mathematical model through mathematisation and reaches a mathematical result by performing its mathematical solution. Transitions to the real result by interpreting these results. The real result is checked by verifying it with real life experiences and the process is completed. Although modelling processes are handled from different perspectives, there is a circularity in all of them. Therefore, if the solution is not convincing, the process continues until the ideal solution is reached (Bukova-Güzel, Tekin-Dede, Hıdıroğlu, Kula-Ünver & Çelik-Özaltun, 2016). Here, the ability of the individual to manage this process and to start the cycle again when he/she cannot reach a satisfactory solution depends on the modelling competence of the individual. Modelling competencies depend on the individual and are expressed as the skills and ability to manage and complete the modelling process in accordance with the purpose (Kaiser & Maaß, 2007). To be able to manage the modelling process, an individual should have social competence (Kaiser, 2007; Kaiser, Schwarz & Tiedemann, 2010), affective competence (Biccard & Wessels, 2011; Maaß, 2006), metacognitive competence (Blum, 2011; Kaiser, 2007; Maaß, 2006) and cognitive competence for mathematisation (Ferri, 2006). The fact that modelling is an argumentative process by nature, and that it is closely related to goals such as making sense, expressing and persuading (Medonça & Justi, 2013) indicates that argumentation skills also play an important role in the mathematical modelling process (Aydın Güç & Kuleyin, 2021).

According to Binkley (1995), argument is the process of forming and presenting predictions. Argumentation is defined as the process of discussing a situation within the framework of contradictory claims and logic (Kuhn, 1993). Argumentation is the process of creating a claim about a problem situation/subject determined by individuals and supporting this claim with the data they collect/observe. The claims on the subject, the data obtained, justifications and rebuttals should be within a certain epistemological framework. Individuals make their defences with explanations supporting their own claims and respond to possible opposing views with these justifications. The structure established in the discussions are guidelines based on cooperative learning principles that help students to complete their research in small groups. In this process, it is very important for individuals to work co-operatively in small or large groups and to communicate effectively and to reflect their own ideas accurately and effectively to the other side. The argumentation process consists of the use of claim, data and justification, the use of rebuttal for the possible counter-case, research on the relevant topic, individual access to information from different sources, interpretation, analysis and evaluation processes (Jimenez-Aleixandre & Erduran, 2007).

There are six components in the argumentation model explained by Toulmin (2003), who explains argumentation as proving and supporting the claims made in science and real life by stating reasons. Figure 2 shows the components of Toulmin's argumentation scheme and the relationship between them.

There are six elements that Toulmin uses as a basis in his framework: Data, claim, justification, support, limitation and rebuttal. Data are the facts used to support the claim and used as evidence. It forms the basis for establishing the discussion. Data can be sample events, statistical information. Claim is an opinion about the value or existing situation. The claim is reached with data. Justification is the reasons, rules, principles that explain the connection between data and claims or conclusions. It explains how the individual evaluates the data and forms a claim (reasoning process). Justifications are of three types: motivational, authoritative and evidential. Motivational justifications are the convictions that the individual uses while defending the claim. Authoritative reasons are the experiences and opinions of the person defending the claim. Supporting are the basic assumptions that validate the specific reason; they strengthen the reason and enable the listeners to understand the reason in the argument. In other words, supports ensure the credibility of the claim. Supports can be personal, exemplary and statistical information such as data. Rebuttals are situations where one of the ideas in the discussion is not valid or statements that weaken the effect of the justification on the claim (Driver et al. 2000; Erduran et al. 2004).

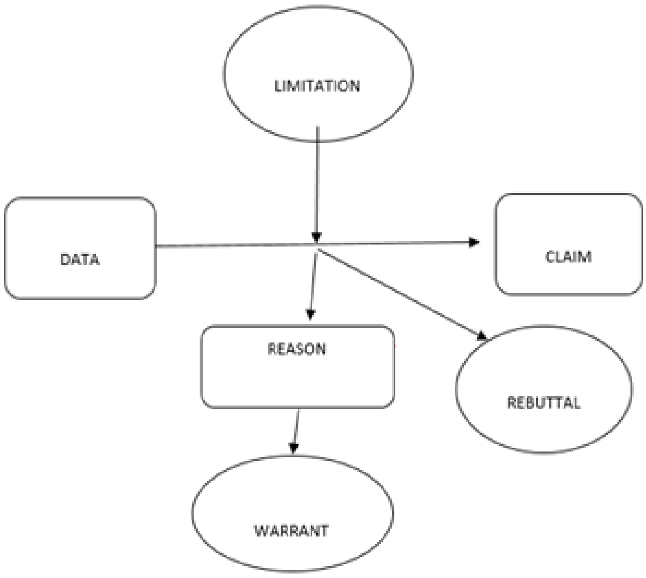


Figure 2: Schematic Representation of an Argument

The basic structure of this model is as in the sentence:

"Because (data).... is (justification)..... in terms of (support)..... therefore (conclusion)"
is as follows.

According to Erduran et al. (2004), Toulmin's discussion model is a model consisting of a claim, data supporting this claim, justifications showing the relationship between the data and the claim, supporters strengthening the justifications, limiters and finally refutations expressing the situations in which the claim is invalid.

Rationale

One of the aims of contemporary education is to enable individuals to understand that society and science are mutually influenced by each other. The concept of Social Scientific Issues (SSI) refers to controversial scientific-based dilemmas that concern society, such as biology, sociology, ethics, ethics, politics, economics and environment (Sadler, 2004). In the early periods of science, science was purer and more limited to the relevant discipline, but in the future, science and society have been integrated into each other and their mutual relationship has gained importance. With this view gaining value, it has become important in science education to raise individuals as individuals who can decide their lives. The use of appropriate methods and techniques, social perspective, creating a discussion environment, students' knowledge of the relevant scientific content and discussing it within the epistemic framework, analysing the relevant subject by thinking in economic, philosophical, ethical and social frameworks are very important elements in the process of addressing SSI in educational processes. The process of addressing SSI requires students to make rational judgements and schools to raise well-informed individuals. SSI are international issues that concern many disciplines and nations. In SSI education, students' critical thinking, relating the issues to daily life and raising individuals as individuals who can cope with these issues come to the fore. SSI are complex, open-ended/open-ended, and controversial issues. Studies on SSI are generally conducted to investigate the nature of science, genetically modified organisms, abortion, global warming etc. and decision-making on these issues and students' reasoning skills (Simonneaux, 2007).

The difficulties encountered in teaching SSI can be summarised as the controversial nature of the topics and the difficulties of bringing together different perspectives to produce arguments. Media and social perspective also play an important role in this process. When addressing SSI, there are goals such as enabling individuals to understand the subject, to contribute to citizenship education, to participate in discussions concerning the society and to understand the nature of science. In the argumentation process, creating arguments and counter-arguments is very important in terms of argument quality. Forming an argument for SSI requires the dimensions of the argument's justifications, how it is handled, its position and logicality, and its multiple perspectives. In this process, the individual may be affected by external factors (media, society, close environment, etc.) in terms of reasoning skills, handling and evaluating the facts he/she believes in. Similarly, the individual may be affected by classroom discussions, multiple perspectives and related disciplines in the argumentation process related to SSI. In this process, the characteristics of the nature of science are also included (Simonneaux, 2007).

The purpose of argumentation is to convince oneself and other participants of the specificity of one's own reasoning (Krummheuer, 1995). Reasoning occurs interactively during mathematical modelling processes (Lesh & Doerr, 2003). Therefore, argumentation, defined as interactions in the observed classroom, relates to the deliberate explanation of the reasoning for a solution, either during or after the development of a solution. It can be a trigger for participants' modelling processes. In this context, this study examines the arguments constructed by the participants in the mathematical modelling cycle and interprets these arguments by considering the modelling processes of the participants.

In the broadest sense, cognitive argumentation is defined as a process of discussion and justification in which more than one person (student or teacher) makes a mathematical claim and provides evidence to support this claim (Conner, et. al. 2014). The cognitive argumentation considered in this study takes into account the arguments of small groups or individuals working on different learning and teaching activities (Conner et. al., 2014; Yackel & Cobb, 1996) rather than examining the arguments of participants during proof activities (Inglis, Mejia-Ramos & Simpson, 2007). The focus is on the interactional aspect of argumentation. To investigate cognitive arguments, Krummheuer (1995) proposes the Toulmin argumentation schema.

When students are engaged in modelling tasks, in transitions between school mathematics and the real world, their cognitive reasoning can be supported in a real-world context during model development. By doing so, they discuss the arguments put forward by individual or group members and try to reach common decisions through their reasoning. In this process they actively formulate arguments and justify these arguments. Therefore, since it is believed that the arguments formed in the modelling cycle are based on modelling processes, it is possible that examining modelling processes can provide insight into students' arguments. Based on the definition of modelling, the mathematical modelling cycle (Blum & Niss, 1991) was chosen as the research framework because it explicitly describes the modelling processes that occur during the modelling cycle. In addition, a second research framework, Toulmin's argumentation schema, was used to examine the structure of arguments in this process (Toulmin, 2003). This study will reveal the arguments within the mathematical modelling cycle by considering the modelling processes based on SBK topics. Considering the focus on mathematical modelling, it is seen that this study has a different context from other cognitive modelling studies due to its argumentative structure. On the other hand, if the focus is on CA, the study is again different from the previous literature in which student arguments are examined during proving and learning/teaching activities due to the emphasis on modelling. In order to uncover arguments in the modelling cycle, the study will ultimately enable to identify the specific aspects that make in-group arguments meaningful, such as how they formulate claims, how they refute each other's claims to arrive at the best joint solution, what this warrant, what claims they make use of and how they arrive at the solution. In line with these aims, the research question of the study is as follows:

"How do participants construct arguments within the mathematical modelling cycle?"

METHOD

Research Design

This research is a qualitative descriptive (exploratory) study aiming to reveal the arguments of pre-service teachers from different departments studying in the second grade in the context of SSI in mathematical modelling practices and how mathematical modelling and argumentation affect each other. Stebbins (2001) defined exploratory research as "a broad-ranging, purposive, systematic, prearranged undertaking designed to maximize the discovery of generalizations leading to description and understanding of an area of social or psychological life. Polit and Beck (2012) briefly consider exploratory research and suggest that it is designed to illuminate how a phenomenon is manifested and is especially useful in uncovering the full nature of a little-understood phenomenon. More specifically, the research seeks answers to the questions: "i) what is the quality of pre-service teachers' arguments in mathematical modelling processes (how/what kind of arguments do they produce), ii) what kind/what kind of arguments emerge in different mathematical modelling practices and how are these arguments connected with argumentation?". For this reason, it is investigated whether pre-service teachers produce arguments in modelling processes according to the determined phenomenon, and if so, how they produce them. In accordance with the purpose of the research, a learning environment was arranged in the learning environment in which students could carry out scientific discussion and modelling practices (2x2 application).

Teaching/Learning Content

While designing the learning environment in which pre-service teachers can present their solution proposals for the related problem situation in the context of SSI, project-based learning (Krajcik et al., 1998), socio-cultural learning environment (Rogoft, 2003) and pre-service teachers' knowledge about how they construct and use models were taken into consideration. Adhering to the theoretical framework, modelling emphasises/requires understanding and explaining the phenomenon of interest and predicting possible changes in the phenomenon. In all lessons, the guiding (main) question was determined as "What solutions would you suggest to reduce/prevent the spread of the Corona Virus (Covid 19)?". In the context of the related question, it is a global and daily problem and the increase in disease around the world and its effects on community health. Based on the problem, the researchers created lesson contents by adhering to the curriculum. After each lesson, practitioners and researchers discussed about the next lesson and content and structured the next lessons.

It was aimed for the prospective teachers to offer solutions by mathematical modelling and argumentation in problem solving in order to reduce spread of the Corona Virus and reduce this disease. The course contents and applications developed by the researchers were carried out with the pre-service teachers. In the implementation processes, two weeks of course content and activities related to mathematical modelling were designed by one of the researchers (R1) before the implementation, and in the next two weeks, courses with argumentation content were designed by the other researcher (R2), and four weeks of modelling and argumentation practices were included in the content of the "Critical and Analytical Thinking" course. Then, the data collection tool developed by the researchers and edited by the subject area expert was applied with the pre-service teachers. The application was carried out for two weeks by researchers 1 and 2, and it was aimed to complete the application by giving research assignments to pre-service teachers when necessary. The aim of the developed data collection tool is to reveal the pre-service teachers' solution generation and decision-making processes by using argumentation and modelling in their processes within the scope of the determined SSI.

Participants

The participant group of the study consisted of 192 (136 female, 56 male; preschool teacher (f=58), primary school teacher (f=52), mathematics teacher (f=40), psychological counselling and guidance teacher candidates (f=42)) pre-service teachers studying in different departments of the faculty of education at a private university in Istanbul. The implementation was designed in accordance with the contents of the "Critical and Analytical Thinking" course that the pre-service teachers took in the spring semester of 2022-2023 and was designed to last for a total of six weeks (four theoretical and two practical courses, online course, average 60 minutes each week). The pre-service teachers had no previous experience in modelling and argumentation. With the activities planned by R1 and R2, theoretical content and activities related to argumentation and modelling were first presented to the pre-service teachers and discussed together, and then a two-week implementation was carried out with the data collection tool developed. During the implementation process, pre-service teachers participated individually, and data was collected in writing while producing their own models and arguments.

Data Collection Process

After four weeks of theoretical content, the researchers prepared a two-week (2x60 min) implementation plan for the actual implementation. Following the content/theoretical knowledge and in-class practices related to modelling and argumentation, pre-service teachers were asked to develop proposals by using argumentation and modelling to offer solutions for the mathematical modelling problem situation developed in the context of the two-week SSI. The data was collected in writing with a data collection tool. In the problem-solving process, the pre-service teachers were left free to search the relevant literature, to construct arguments and to make modelling in the process of proposing solutions. Each pre-service teacher was asked to construct his/her own model individually and to record his/her arguments in writing.

Data Collection Tool

Socio-scientific scenarios

A scenario involving a global problem (Corona Virus/Covid19) was developed by the researchers in order to enable pre-service teachers to produce socio-scientific arguments by using modelling for the problem. The scenario was prepared in a way to attract students' attention by presenting examples from daily life and to benefit from mathematical modelling in the solution process. For the developed scenario, expert opinions were taken in the fields of Turkish, socio-scientific issues and argumentation. As a result of the feedback received, the scenarios were made ready for implementation. The necessary ethics committee permissions were obtained before starting the applications.

Data Analyses

Considering the purpose of the study and the phenomena used to achieve this purpose, data analysis was carried out in two stages. The primary focus of data analysis was on the components of mathematical modelling. In this context, first of all, the transcription text was examined according to its modelling components and the words in the transcription were considered as the unit of analysis. In performing the analysis, as Conner et al. (2014) did, the researchers first focused on the mathematization processes in discourse to identify the discussion sections. In the second stage, the focus was on the argumentation schemes that were formed during their mathematization processes and then put forward to persuade the other party with their ideas based on mathematical evidence.

Stage 1: Modelling

After identifying all episodes involving modelling practices (step 1), we coded the cognitive processes involved in students' performance of these modelling phases (Sins et al., 2009). With the awareness that the modelling process is a challenging learning task that requires specific cognitive processes of different difficulties (Hogan & Thomas, 2001; Löhner et al., 2005; Sins et al., 2009). Cognitive strategies for the modelling framework were developed based on empirical evidence (see Table 2. This suggests that the majority of learners use the following cognitive strategies during modelling: (a) analysing objects and factors to be included in their models, (b) reasoning about the relationships of their factors, (c) synthesising a model using a model, (d) trying to explain relationships, and (e) testing their models, albeit in different ways and depth. The categories mentioned above were used as a guiding framework for students' modelling analyses.

Table 1. Cognitive Strategies during Modelling

Cognitive Process	Indicator
Analyse: Students talk about the elements of modelling. They identify factors that may or may not be relevant to their model. They identify elements of the phenomenon that should be included in their model.	**Participant 17:** A lot of data was given in the article about COVID-19 testing that I read. I will use this data in my mathematical model because it contains information about many patients, this data will help me reach the real model. Realization of the role of decision makers regarding COVID-19, Determination of variables related to mathematical change for the briefing that can be given to the doctor
Inductive reasoning: Learners elaborate elements within or relative to their model (mainly involving qualitative reasoning). They express hypotheses about how the elements interact and how they should behave in their model.	**Participant 13:** COVID-19 testing is not always accurate. But the probability of giving correct results is much higher than not giving correct results. The mathematical model or proof of the balance between the reliability of the test and finding the right result, determining which variables the ideas add or remove from the model, deciding on the variables of the mathematical model
Explain: Students explain how elements of their model work or why they are included (or excluded) with mathematical evidence.	**Participant 2:** We decided to compare our patient's test results with others. In this way, we will be able to come to a conclusion. You can see them here [on the model]. Determining the status of the variables determined in the model, expressing the objective function,
Evaluate: Students test whether their data (phenomena) and their model work. They determine whether the model explains the data. To do this, students revise their model.	**Participant 19:** Our mathematical model gives accurate results, but we need to apply it to other patients to determine that the test is truly reliable. Prove or disprove the function in the model by operation. Description of the real-world equivalent of the mathematical result.

Stage 2: Argumentation

In argument analysis, the relevant content is analysed and examined in line with a certain epistemological framework. Certain epistemological competences are required to create high quality arguments. Because students participate in the process with indirect experiences. However, in some cases, although students have sufficient data, they may still make wrong interpretations. Therefore, it is important to consider field-dependent criteria in argument analysis (Erduran, 2007).

In the study, the qualitative data from the argumentation and modelling process were analysed by taking into account the methodological tool developed by Erduran et al. (2004) and Toulmin (Jimenez-Aleixandre & Erduran, 2007) argumentation models. The qualitative data collected from the discussions of each participant with the help of the specified scenario were categorised into levels according to the tool used. In the socio-scientific argumentation process, the levels of the argumentations made by the individuals in the groups and the number of these levels were determined (LEVEL 1= 1 point, LEVEL 2= 2 points, LEVEL 3= 3 points, LEVEL 4= 4 points, LEVEL 5= 5 points).

Table 2. the Levels of the Arguments

Level	Indicator
1	No evidence
2	There is scientific evidence; there is diversity (type of evidence).
3	There is high-level thinking (analytical, creative, critical, systemic, etc.).
4	There is scientific evidence, there is diversity (type of evidence), there is high-level thinking (analytical, creative, critical, systemic, etc.).
5	There is scientific evidence, there is diversity (type of evidence), there is high-level thinking (analytical, creative, critical, systemic, etc.), there is the ability to reflect all possibilities in a whole.

The basic structure of this model is as in the sentence:

"Because (data).... is (justification)..... in terms of (support)..... therefore (conclusion)".

According to Erduran et al. (2004), Toulmin's discussion model is a model consisting of a claim, data supporting this claim, reasons showing the relationship between the data and the claim, supporters strengthening the reasons, limiters and finally refutations expressing the situations in which the claim is invalid. The first of these components is the claim, the step in which the individual puts forward his/her claim. The second component, data, is the stage in which the individual presents the data supporting his/her claim, and the last of the basic components, justification, is the stage in which he/she establishes a relationship between the data and the claim. Apart from these three basic components, supporters, qualifiers and rebuttals are auxiliary components of the model. Supporters are the statements put forward to strengthen the justification, qualifiers are the statements that show the effect of the justification on the claim, and rebuttals are the statements that weaken the effect of the justification on the claim.

FINDINGS

The findings of the study will examine the sub-arguments constructed within the modelling cycle and then their interpretation by considering the modelling processes in response to the research question.

The pre-service teachers were given a task belonging to the context at the center of their lives. This context is a mathematical modelling activity related to the COVID-19 pandemic. After reading the task, pre-service teachers were expected to first produce two questions about the COVID-19 epidemic and the virus that caused it.

In the second stage, they were asked to explain why they produced these questions and asked them with a purpose. The aim here is to enable them to determine their arguments on COVID-19 through the questions they produce.

In the third stage, the routine procedure of how to diagnose a person with coronavirus is explained. Then, the data of a patient who came to the hospital was shared with them and they were asked to explain statistically whether that person had COVID-19, based on a mathematical model. Questions asked at this stage;

a. What is the probability that a person with a positive PCR test really has COVID-19?

b. Write a letter to the doctor describing the method you created to find this ratio.

Based on their different real-life experiences, pre-service teachers tried to demonstrate the mathematical modelling of a PCR test related to the COVID-19 pandemic, and the sub-arguments they developed during the modelling process were revealed. In this process, pre-service teachers were asked to write a report to the doctor in the event network in order to detail their arguments and to be able to use their mathematical models in creating arguments or to determine that they support their arguments and models.

The mathematical model desired to be created here needs to be built on their understanding of the COVID-19 article and how the PCR test is done. They are then expected to construct a statistical model as expressed in the task. In the construction of this model, it was necessary to pay attention to the article and determine how the safety of the PCR test was regulated. Therefore, since there are two different variables, Bayes' theorem, one of the statistical models, is included.

Participants tried to determine whether the person who came to the hospital had COVID-19 by using PCR test results and other information about the person. They tried to make a claim based on the available data. To arrive at this claim, they used the information and statistical data contained in the task paper to develop the argument. They also ignored the claim that a person was 100% COVID, even if the PCR test was positive, as refuting. They did not feel the need to justify this sub-argument as they made a claim based on direct data. When the above-mentioned solution is considered in terms of the mathematical modelling process, the statistical information given in the task has become the real models used by the participants. Based on the real-life experiences of the participants and the concrete data they used, they assumed that the probability of a person with a positive PCR test to be COVID should be supported with other data, and they focused on building the mathematical model of the task based on these real models and made some calculations. They were able to construct sub-argument schemes in which the drawing of a mathematical model based on their assumptions became the assertion.

According to the results of the preliminary analysis, the focus of this study is the arguments of the participants, as the study examines the arguments generated within the modelling cycle. In this context, different sub-arguments of the participants emerged throughout the modelling cycle.

The first important result of the study is that most sub-arguments are combined with data assertions. This is in line with studies that have found that the sub-arguments are linked to the data claim (Krummheuer, 1995; Conner et. al., 2014).

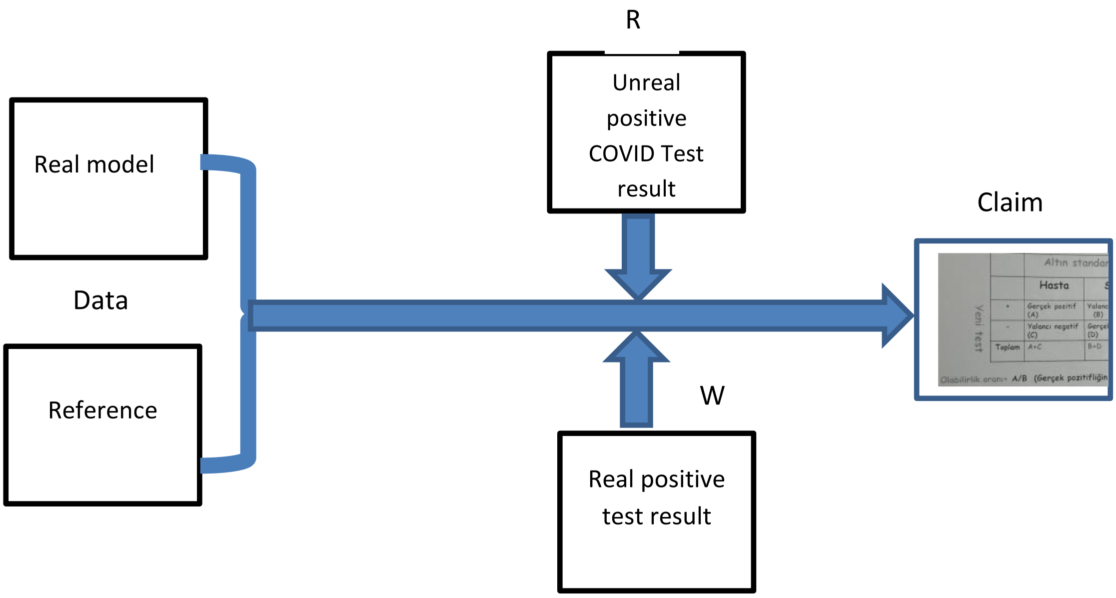
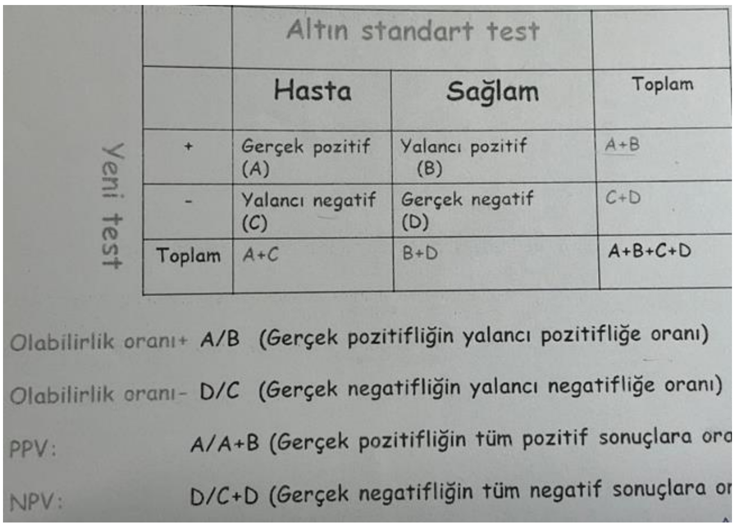


Figure 3. Schematic Representation of an Teacher Candidates' Argument



		Altın standart test		
		Hasta	Sağlam	Toplam
Yeni test	+	Gerçek pozitif (A)	Yalancı pozitif (B)	A+B
	-	Yalancı negatif (C)	Gerçek negatif (D)	C+D
	Toplam	A+C	B+D	A+B+C+D

Olabilirlik oranı+ A/B (Gerçek pozitifliğin yalancı pozitifliğe oranı)
Olabilirlik oranı- D/C (Gerçek negatifliğin yalancı negatifliğe oranı)
PPV: A/A+B (Gerçek pozitifliğin tüm pozitif sonuçlara ora
NPV: D/C+D (Gerçek negatifliğin tüm negatif sonuçlara or

Figure 4. An example for Teacher Candidates' Explanations

The participant expressed this visual as the basis of his mathematical model. As a result of his research, he tried to explain the mathematical model he tried to build by adding two new concepts (sensitivity and specificity) to the data given in the activity. In the reasoning process to support his arguments, he found the total positive test result by adding the numbers of 80%, which is the probability of PCR tests being positive, and 9.69%, which is false positive. He tried to make a proportion with the positive scale of the whole PCR test but made an error in the reasoning. This type of error was procedural. In the process of explaining the mathematical model he constructed, instead of calculating the probability of the PCR test result being correct by putting forward a simpler reasoning, that is, instead of operating the model, he made operations based on the reasoning he first constructed and exhibited another error by equating the result to the total positive rate of the PCR test. As a result of the incorrect procedures, he could not provide any evidence for the last step of modelling, which is evaluation.

Table 3. Categorisation and Descriptions of Arguments

Modelling	Description
Model construction	Students construct their model to account for the phenomenon (COVID-19) under study
Model use	Students use their model with an aim to predict the evolution of the phenomenon and decide upon the most suitable solution to the problem (Chance of contracting the virus)
Model evaluation (and revision)	To submit a report to the doctor to evaluate whether the model he/she has created is valid.

Table 4. Example for Teacher Candidates' Arguments and Explanations

Category	Example
Evidence: Evidence is scientific data that supports the claim. The evidence can come from investigations students engage in firsthand or from research conducted online or in books that provide data	I can use a math formula to determine if the test is real. With this formula, the rate of the test actually being positive was the same as in the article (Participant 17)
Reasoning: Reasoning provides a justification for why or how the evidence supports the claim	(My mathematical model proves) People without COVID also test positive, but the opposite situation also gives erroneous results in the test. We cannot operate with my formula without finding the error rates. (Participant 13)
Rebuttal describes an alternative claim and provides counterevidence and counter reasoning for why the alternative claim is not appropriate.	Participant 13: the According to the test, our 40-year-old patient has COVID. participant14: Our patient does not have COVID, my results show a 17% probability of him having it.... participant17: Maybe not, according to my model, the result is 39%...

The arguments used by participants who can put forward mathematical models while constructing, using and explaining (proving) these models are presented together. This relationship can be seen in Table 5. Here, the mathematical model processes and argument types put forward by the participants are compared.

Table 5. The Use of Evidence, Reasoning and Rebuttal

	Modelling Construction	Modelling Use	Modelling Evaluate	Total
Evidence	68	84	7	159
Reasoning		6		6
Rebuttal	1	3	18	22

As seen in Table 5, students discussed their proofs only by using their own models. On the contrary, the use of reasoning and rebuttals occurred with or without the use of their models. In order to investigate how students used the model during argumentation and how they entered into argumentation without using their models, the issue is analysed in detail below.

Modelling Construction, the following excerpt is the first step of the participants' constructing their own mathematical models during the model building practice and explaining these models with a mathematical system to reveal the infection status of the virus. Accordingly, it is an example of how they explained about the mathematical formula system (for example, finding a result by associating the numbers given in the problem situation).

> Participant 16: From a mathematical point of view, I first focused on the rate at which the test could identify a true positive patient. The data in the article stated that this was shown with 80% accuracy. I found a value of 0.80*0.01=0.008, which would mean that a test is reliable. I think the margin of error is 8 per thousand.

It shows how participants used evidence to support why a particular solution was appropriate for their problem when creating their model. The participant researched sources for the solution of the problem and identified algorithms for patients and non-patients. He entered the numerical values given by the researchers in the problem into this algorithm and completed the model construction. In this process, she exhibited arguments for using sources and associating evidence with sources.

Finally, during the model building process, participants provided rebuttals about the possible solution of the problem. For example, a group of participants refuted their own solutions by finding that there are different ways to calculate the probability of a false result of the COVID-19 test.

Modelling Use, Our analysis (Table 5) also shows that the argumentation took place through the use of modelling. For example, participants reasoned about the consequences of their solutions or rebutted about the solutions and their feasibility without actually using their models, despite having their mathematical models in front of them. The following quote is a representative example of how the participant questioned the feasibility of the solution proposed by another source:

> Participant 16: If I can find a similar ratio between true positives and false positives, I can mathematically determine the true value of the test... What is defined as the likelihood ratio is shown as the ratio of true positives to false positives. For me this calculation is 0.80/0.01=80...

It was also observed that the participants reasoned and discussed about the consequences of the solutions proposed without using the model. Some participants ignored the possibility of the COVID-19 test giving incorrect results and did not reflect it in their mathematical models. This turned out to be a critical mistake in their use of mathematical models, which affected both the outcome and the next step of mathematical modelling: evaluation.

Modelling Evaluate, in the last step of mathematical modelling, the evaluation step, it was determined that the participants could not provide satisfactory information. The frequencies expressed in Table 5 support this situation. While engaging in the cognitive process of explanation, the participants were also engaged in defending a knowledge-based claim, explaining or predicting a fact, and using knowledge. However, almost the majority of the participants could not provide satisfactory data at the explanation-evaluation stage because they made critical errors in the process of constructing a model and coming up with an idea using this model.

Controversies in the Cognitive Processes of Modelling, As mentioned in the theoretical framework, it has been confirmed that different cognitive processes occur during modelling. For the explanation of the categories, we used the cognitive processes defined by Sins et al. (2009), namely analysing, evaluating and explaining the model and engaging in inductive reasoning. Based on our analysis of the cognitive processes and the open coding of the discussions that took place during these processes, it emerged that when students were engaged in argumentation and modelling, they practiced the following actions: (a) cited claims, (b) defended a claim, (c) explained, and (d) asked for information. How argumentation is enacted in specific cognitive processes of mathematical modelling,

The analysis of the participants' argumentation and modelling presented above shows that the participants requested information and asked questions to help them develop their models and at the same time develop their arguments. It is clear the model was a visual aid in understanding the information given in the problem situation, especially in understanding the participants' calculation and interpretation of the accuracy value of the disease tests. In addition, the participants who created mathematical models by identifying models, solutions and limitations were able to engage in higher level argumentation involving reasoning and rebuttal. In both sub-arguments, their rebuttals simplified the problem so that they could solve it. Therefore, in this exercise, participants provided rebuttals based on their assumptions. The three qualifiers used by the participants indicated approximate values. In other words, while constructing the mathematical model, they reached an approximate value or situation based on the rebuttal assumption, and while making assumptions, they interpreted the values given in the problem according to the desired results. They had difficulty in coming up with an original mathematical model and used qualifiers to reach the actual result.

CONCLUSION and DISCUSSION

The argument structures (Johnson & Blair, 2006) of pre-service teachers studying in different departments for mathematical modelling and the problem situation explaining it (Johnson & Blair, 2006) were analysed while proposing solutions for a problem situation involving socio-scientific issues. In the findings obtained, Toulmin's argumentation model (Toulmin, 1958) was used to analyse the argument structures of pre-service teachers towards covid 19. The argument levels of pre-service teachers were analysed by classifying them according to 5 different levels. Level 1 was categorised as the lowest level and Level 5 as the highest level. When the argument structures used by the pre-service teachers in their solution proposals and modelling processes were analysed in detail, it was seen that there was an accumulation mostly at Level 2, and there were not many models at Level 3 and Level 4. It was found that the pre-service teachers grouped at Level 2 focused on evidence-based reasoning and the diversity of evidence when constructing arguments or proposing a solution to a scientific text, that this evidence should be obtained by the scientific method, that it should be published by experts in the subject area, and that they reached conclusions by diversifying the examples they presented. They emphasised that the evidence should have high persuasive power and scientific quality (Çekbaş, 2017). When the argument structures classified as level 3 were examined, it was found that the pre-service teachers mostly established cause and effect relationships, looked at the events from a critical point of view, and although the starting point of the model put forward was based on a scientific fact, they formed faulty structures while constructing the model (Johnson & Blair, 2006). It has been concluded that the models established with the lack of subject area knowledge, misconstruction or faulty courts do not explain the relationships, that they cannot answer the problem when evaluated scientifically, that the model they established is generally established by deductive reasoning, and that the models and arguments close to the model in the example presented and these arguments are concluded by the supporters of different evidence. It was revealed that the explanations put forward that the reasoning and arguments were structured incorrectly did not provide reality when the steps put forward in the model were advanced. No data were found for the arguments and modelling of pre-service teachers classified as Level 4 and Level 5. In modelling, it was determined that there were erroneous structures encountered in Level 2 and Level 3.

RECOMMENDATIONS

The findings obtained at the end of the study showed that pre-service teachers participated in modelling and argumentation processes, but they encountered some barriers at the points of presenting solution proposals and proving the models and their argument levels were low. In order to raise these levels to higher levels, SSI-contextual problems and course contents in which they can present solutions applicable to the problem can be developed. In this way, pre-service teachers can gain concrete experience in the development of modelling and argumentation skills.

Curriculum developers and policy makers can work together to add teaching profession courses that can develop modelling and argumentation skills to teacher training programmes, and the process can be carried out together with field experts. Workshops and teaching activities can be planned to support the modelling and argumentation skills of pre-service teachers together with field experts.

ETHICAL TEXT

"In this article, the journal writing rules, publication principles, research and publication ethics, and journal ethical rules were followed. The responsibility belongs to the author (s) for any violations that may arise regarding the article. This study was approved by the decision numbered 2022/6 meeting dated 28.07.2022 in line with the decisions of the Scientific Research and Publication Ethics Committee of Istanbul Aydin University."

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