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AN INVESTIGATION ON THE ALIGNMENT OF PROSPECTIVE MATHEMATICS TEACHERS' RESPONDING TO STUDENT THINKING IN THEORY AND APPROXIMATION OF PRACTICE

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ABSTRACT

Teachers' ability to respond to student thinking is important for effective teaching. Considering the importance of the decisions and steps to be taken by teachers in responding to student thinking in teaching environments where conceptual understanding that brings meaningful learning is aimed at, it is understood that prospective teachers' skills in responding to student thinking should be improved. One of the current approximation of practices used in the development of prospective teachers' instructional skills is scripting task practices. The aim of this study is to reveal the alignment between theory and approximation of practice in the context of conceptual and procedural understanding of prospective mathematics teachers' ability to respond to student thinking through scripting tasks. Being one of the qualitative research methods, in the present study, we used a case study to examine how prospective teachers responded to students' ideas. For this purpose, we asked 38 prospective mathematics teachers to write how they would take an instructional step on a problem related to student thinking and then justify it by completing a scenario prepared on the same problem. We subjected the data to content analysis and open coding. The findings revealed that nearly half of the prospective teachers aspired to teach for conceptual understanding and were able to demonstrate this in their scripting task practices. There were prospective teachers who aimed for conceptual teaching in theory, but could not reflect this focus in their approximation of practices. There were also prospective teachers who pursued only a procedural focus, both in theory and in approximation of practice. There were no prospective teachers who pursued a procedural focus in theory and realized conceptual teaching as an approximation of practice. We discuss the findings in the context of the relevant literature.

Keywords: Scripting tasks, responding, conceptual understanding.

INTRODUCTION

Since 2006, following the reform movement in education in Turkiye, expectations from teachers have evolved in a direction that aims to ensure student-centered teaching and conceptual understanding. With the reform movement in the mathematics curriculum, teachers are expected to plan lessons in active learning environments where students are at the center and can build their conceptual understanding (Ministry of National Education [MoNE], 2018). These expectations are directly related to prospective teachers who are the teachers of the future. During their undergraduate education, prospective teachers who plan and implement lessons in which they can put students at the center with the constructivist approach and in which students can achieve meaningful learning through the use of multiple representations and active discussion environments are trying to be prepared for reform-based teaching. The fact that prospective teachers have limited opportunities for approximation of practice within the scope of the Teaching Practicum course they take in their senior year, apart from the theoretical courses they take during their undergraduate education in order to adapt to the reform movement, makes every effort to increase these opportunities important. At this point, the use of scripting tasks in initial teacher education draws attention.

Scripting Tasks as Approximation of Practices in Teacher Education

One of the current approximation of practices used in the development of prospective teachers' instructional skills is scripting task practices. In scripting task practices, which are considered as one of the practice-based pedagogies, prospective teachers are presented with a classroom scenario situation and asked to complete the dialog in the given scenario. Meanwhile, prospective teachers reveal the students they visualize in their minds (Zazkis & Herbst, 2018) and what kind of decisions they will make in the classroom environment. While doing this, they also use their knowledge about student thinking. In the scenario completion process, prospective teachers try to make sense of student ideas, ask questions to the students they visualize in their minds, establish dialogues among students, use materials and models for teaching, and respond to student thinking (Campbell & Baldinger, 2022). With these practices, it is aimed to reveal the instructional approximation of practices of prospective teachers (Campbell & Baldinger, 2022). It is stated that the use of scripting task applications in teacher education can provide instructional support to prospective teachers (Bergman et al., 2023; Shure & Liljedahl, 2023). One advantage of scripting task applications is that prospective teachers can focus on teaching and student thinking in an environment isolated from factors such as classroom management (Grossman et al., 2009). With scripting task practices, prospective teachers have the opportunity to respond to student thinking before they move to the real classroom environment (Bergman et al., 2023).

As the findings of Shure and Liljedahl's (2023) study reveal, prospective mathematics teachers can exhibit a teacher-centered structure and a teaching that is far from the focus on conceptual understanding. At this point, the purpose of this study is to examine the alignment between the teaching moves that prospective mathematics teachers advocate in theory and the moves they exhibit in their approximation of practices through the scripting task application. It is thought to contribute to the literature by revealing how prospective mathematics teachers plan to respond to student thinking and to what extent they can reflect this in their approximation of practices.

Responding to Student Thinking

Teachers' ability to respond to student thinking is important for effective teaching (Monson et al., 2020). Bergman et al. (2023) underline the necessity for teachers to have the ability to respond to student thinking. As Beswick (2020) points out, the way teachers respond to student thinking is influenced by their perspective on teaching. In-service teachers/prospective teachers who believe that lessons in which student ideas are listened to and taken as a basis, the lesson flow is shaped by taking student ideas into account, and conceptual understanding is targeted are effective, are expected to act accordingly in their instructional approximation of practices. Teachers who focus on interpreting and responding to student thinking (Jacobs et al., 2010) focus on student ideas in their lessons and determine their next instructional steps accordingly, thus aiming to develop students' reasoning (Ivars et al., 2020; Land et al., 2019).

Instructional environments where the teacher is the answer center are not compatible with meaningful learning goals (Monson et al., 2020). On the other hand, teachers may find it difficult to take the right instructional steps to elicit and monitor student thinking (Franke et al., 2009). In a study by Kabadaş and Yavuz Mumcu (2024), it was found that in the process of identifying students' misconceptions, teachers often failed to focus sufficiently on students' thinking and dealt with students' mistakes superficially; in the process of eliminating the misconceptions, they told students that their answers were wrong and could not make students realize their own mistakes. As underlined by Son (2013), prospective teachers may exhibit a procedural approach when responding to student errors. Considering the importance of the decisions and steps to be taken by teachers in responding to student thinking in teaching environments where conceptual understanding that brings meaningful learning is aimed at, it is understood that prospective teachers' skills in responding to student thinking should be improved. At this point, there is a need for professional development environments where student thinking can be analyzed (Schack et al., 2013). As stated by Son (2013), prospective teachers should be provided with environments that will enable them to approximate the practice of responding to both conceptual and procedural student errors.

In the present study, we focused on the measurement area to understand prospective teachers’ responding to student thinking through misconceptions. Measurement is crucial for the instruction of numerous mathematics subjects, including algebra, numbers, and geometry (Reys et al., 2001). According to a variety of studies in the literature, students encounter difficulties comprehending the concept, and perform a greater number of measurement errors, particularly in the areas of length, area, and volume, than in other areas of mathematics (Chappell & Thompson, 1999).

In the light of the above-mentioned literature, this study aims to examine the alignment of theory and approximation of practice in the context of conceptual and procedural understanding of prospective middle school mathematics teachers' ability to respond to student thinking through scripting tasks. The research questions of the study are as follows: 1) How do prospective teachers plan to respond to student thinking in theory? 2) How do prospective teachers respond to student thinking in approximation of practice through scripting tasks? 3) To what extent is there an alignment between prospective teachers' skills in responding to student thinking in theory and approximation of practice?

METHOD

This study, which aims to reveal the alignment between prospective mathematics teachers' ability to respond to student thinking in theory and approximation of practice in the context of conceptual and procedural understanding, is qualitative in nature. Qualitative studies aim to understand what happens in a particular context (Patton, 1985) and this study aims to understand how prospective mathematics teachers respond to student thinking in theory and approximation of practice within the context of a course. We specifically used a case study, one of the qualitative research methods. Case studies are used to find details, provide explanations, and thoroughly explore a situation in order to answer the questions of what, how, and why the study is being conducted (Yin, 2003). The present study examined how prospective teachers responded to students' ideas. We thoroughly reviewed their answers, rationales, and instructional recommendations, and applied the case study methodology.

Participants

This study was conducted with 38 out of a total of 47 prospective teachers who participated in the Misconceptions in Mathematics Teaching course at the senior level of the Department of Elementary Mathematics Education in the fall semester at a state university in Turkiye. These participants were the prospective teachers who gave full responses to the data collection tools.

The duration of education in the Department of Elementary Mathematics Teacher Education is four years and prospective teachers are trained to teach students aged 11-14 at the middle school level in accordance with the constructivist approach, student-centered and meaningful learning. The participating prospective teachers in this study are senior level prospective teachers who have completed most of their theoretical courses and have the opportunity to put into approximation of practice what they have learned in theory with the Teaching Practicum course. The participants were selected from among the prospective mathematics teachers (PMTs) who took the 14-week compulsory course on Misconceptions in Mathematics Teaching. In this course, PMTs are given theoretical knowledge about misconceptions that students encounter in mathematics as well as the opportunity to approximation of practice through scripting task applications. The ethical permissions required within the scope of the study were obtained by Trakya University Social and Human Sciences Research Ethics Committee with the decision numbered E-29563864-050.04.04.04-368280 dated 07.12.2022, and prospective teachers voluntarily participated in the study.

Data Collection and Procedure

The data collection tools of the study consisted of an open-ended question that was distributed to the participants in the first week of the course and directed to them individually through a problem situation about student thinking. Then, a scenario was completed individually during the course. These data collection tools were prepared by the researchers in the light of the relevant literature and finalized after expert opinion was taken.

The open-ended question included the following problem situation and student response (see Figure 1) that were about calculating the volume a rectangular right prism. The open-ended question was given to the participants at the beginning of the semester. The student response consisted of the misconception about not adding the invisible faces to the volume calculation (Battista & Clements, 1996; Tan Sisman & Aksu, 2016). Accordingly, we posed the below open-ended question:

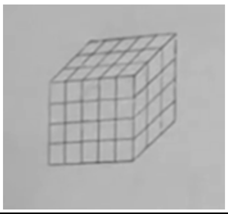
Problem situation:	Calculate the volume of the rectangular prism below. 
Student answer:	To find the volume of a rectangular prism, I need to count the cubes. There are 20 cubes of 5x4 on the front side. There are 12 cubes of 3x4 cubes on this side. When I look at the top face, there are 15 cubes of 5 by 3. Since these are also on the opposite sides, I need to multiply the sum 20 + 12 + 15 by 2. 47 x 2 gives 94.
Written interview question:	If there is a misconception in the given student answer, explain what you would suggest/do as a teacher to eliminate it.

Figure 1. The Open-Ended Question

In the scripting task application, which constitutes the second data collection tool, the participants were given the following scenario and class discussion situation (see Figure 2). We asked the participants to put themselves in the place of the teacher in the scenario and to continue the class discussion at the length they wished in the places left dotted, and also to justify and explain why they continued the discussion in that way.

Scenario	Classroom discussion	Questions
In this scenario, the teacher asks students to find the volume of the rectangular prism below. Students share their ideas with the class. 	Teacher: How did you calculate the volume? Who wants to explain? Nazlı: The answer is 52 cubic units. Teacher: How did you come to this conclusion Nazlı? Can you explain to the class? Nazlı: To find the volume of a rectangular prism, we need to find the total number of cubes. There are 12 cubes from 4 x 3 on the front side. There are 6 cubes of 2 x 3 on the side. The top face has 8 cubes of 4 x 2. Since there are faces directly opposite these faces, we make 2 x (12 + 6 + 8).	Put yourself in this teacher's shoes and move the class discussion forward where dots are left out. Why did you move the discussion forward in this way? Explain your reasons.

Figure 2. Scripting Task Application

Data Analysis

We used content analysis for the data analysis. We used open coding first, in order to answer the first research question. We coded the PMTs' answers to the open-ended question. At this stage, the open codes of using concrete models and technology, asking questions, explaining reasoning, providing solution/formula were emerged. In order to answer the second research question, we examined PMTs' completed scenarios and used open coding together with their justifications. At this stage, the codes of comprehension through model/technology, asking questions, explaining reasoning, making the student notice his/her mistake, telling him/her that he/she is wrong, telling formula, and student's enlightenment were emerged. For answering the third research question, we investigated the alignment between PMTs' answers in the open-ended question (theory) and skills of responding to student thinking (approximation of practice). First, the possibilities of the alignments were grouped as consistency/inconsistency. Then, the data were coded as conceptual-conceptual, procedural-procedural under the consistency group; and coded as conceptual-procedural, and procedural-conceptual under the inconsistency group.

In the data analysis phase, all data were first coded separately by both researchers. Then the researchers came together and compared their codes. Using the formula created by Miles and Huberman (1994), inter-coder reliability was first calculated and found to be 92%. A rate greater than 70% suggests the study's reliability (Miles & Huberman, 1994). Discrepancies in the coding were resolved by discussing, and eventually full consensus was reached. In order to ensure the validity and reliability of the study, direct quotations from each data type are included in the findings section.

FINDINGS

In this section, we discussed the findings under sub-headings related research questions, respectively.

PMTs' Planning to Respond to Student Thinking in Theory

Analyzing the PMTs' answers to the open-ended question revealed that the majority (68%) intended to teach *using concrete models and technology*. This was followed by the code of *making the student notice his/her mistakes and/or explaining the reasoning of the operation to the student*, which accounted for 26% of the responses. We observed that only 18% of PMTs planned to *tell the answer directly to the student or make the student perform the operation by giving the formula*. Very few PMTs planned to teach the lesson by *asking questions*. Table 1 below presents the codes indicating PMTs' plans to respond to student thinking in theory, along with their respective frequencies and percentages.

Table 1. Frequencies and Percentages of Responding to Student Thinking in Theory

Codes	Frequencies (percentages)*
Using concrete models and technology	26 (68%)
Making student notice his/her mistake and/or explaining reasoning of the operation to the student	10 (26%)
Telling answer directly to the student/ making the student perform the operation by giving the formula	7 (18%)
Asking questions	2 (5%)

*Since there was more than one coding for some PMTs, the sum of the percentages exceeded 100.

PMTs' Responding to Student Thinking in Approximation of Practice through Scripting Tasks

After analyzing the data from the PMTs' scripting task applications, we observed that half of them (50%) adopted the approach of *telling the answer or formula and using it*. The instructional approach of *making the student notice his/her mistake and explaining the reasoning of the operation to the student* trailed closely behind, accounting for 42%. Only 32% of PMTs approximated their practice *using concrete models and technology*. There were also PMTs who *told the student that he/she was wrong* (21%), as well as PMTs who *asked questions* to the student (18%) and then waited for *student enlightenment* (16%). Table 2 presents the codes indicating how the PMTs responded to student thinking in an approximation of practice, along with their frequencies and percentages.

Table 2. Frequencies and Percentages of Responding to Student Thinking in Approximation of Practice

Codes	Frequencies (percentages)*
Telling answer/formula and using the formula	19 (50%)
Making the student notice his/her mistake and/or Explaining the reasoning of the operation to the student	16 (42%)
Using concrete models and technology	12 (32%)
Telling student that he/she was wrong	8 (21%)
Asking questions	7 (18%)
Student enlightenment	6 (16%)

*Since there was more than one coding for some PMTs, the sum of the percentages exceeded 100.

Alignment between PMTs' Responding to Student Thinking in Theory and Approximation of Practice

Regarding the alignment between theory and approximation of practice, the findings show that 17 out of 38 PMTs aimed to teach towards conceptual understanding with the highest percentage and were able to demonstrate this in scripting task applications (45%). However, this percentage was below 50% of the participants. PMTs, who aimed for conceptual teaching in theory but could not reflect this in their approximation of practices, accounted for 31%, indicating a misalignment between theory and practice. There were also PMTs, with a rate of 24%, who pursued only a procedural focus in both theory and approximation of practice. There were no PMTs who pursued a procedural focus in theory and realized conceptual teaching as an approximation of practice. Table 3 below presents the codes indicating the extent of alignment between PMTs' ability to respond to student thinking in theory and approximation of practice, along with their respective frequencies and percentages.

Table 3. Frequencies and Percentages of Alignment between Theory and Approximation of Practice

Theory-approximation of practice alignment	Theory	Approximation of practice	Frequency (percentage)
Consistency	Conceptual	Conceptual	17 (45%)
	Procedural	Procedural	9 (24%)
Inconsistency	Conceptual	Procedural	12 (31%)
	Procedural	Conceptual	0 (0%)

The findings also indicate that most of the PMTs (26 PMTs, 68%) who aimed to teach for conceptual understanding in theory aimed to achieve this through the *use of concrete models and technology*. Parallel to this, in approximation of practice, 17 (45%) of the PMTs who aimed for conceptual understanding and three (8%) of those who focused on procedural understanding used concrete models.

Below, the findings on the alignment between theory and approximation of practice is detailed under related sub-titles.

Consistency of Theory-Approximation of Practice

To illustrate the findings, PMT6, a prospective teacher who aimed and implemented teaching towards conceptual understanding by demonstrating alignment between theory and approximation of practice, provided the following response to the open-ended question in the given problem scenario: "*To overcome this misconception, I would prefer to use concrete material. This way, the student would have seen clearly where he/she made a mistake.*"

We observed that PMT6 finished the scenario concurrently with this response as below:

> *Teacher: How did you calculate the volume? Who wants to explain?*
> *Nazlı: The answer is 52 cubic units.*
> *Teacher: How did you come to this conclusion Nazlı? Can you explain to the class?*
> *Nazlı: To find the volume of a rectangular prism, you need to find the total number of cubes. There are 12 cubes from 4 x 3 on the front side. There are 6 cubes of 2 x 3 on the side. The top face has 8 cubes of 4 x 2. Since there are faces directly opposite these faces, we make 2 x (12 + 6 + 8).*
> *(Rubik's cube is brought to the classroom and students are given the opportunity to examine it).*
> *Teacher: Okay Nazlı, you said that there are 12 cubes on the front side, let's mark those cubes in the figure and discard them. Then wouldn't some face from the sides and top be discarded as well?*
> *Nazlı: Oh yes, teacher. Then there are 12 cubes on the front, 12 on the back, 24 cubes in total.*
> *Teacher: Did you realize your mistake, Nazlı?*
> *Nazlı: Yes, teacher.*

PMT6 justified the completion of the scenario above by stating that "*It is very difficult for students to perceive three-dimensional objects with all their aspects. Using concrete materials as much as possible will help us a lot in terms of concretization."*

Focusing on conceptual understanding in theory and approximation of practice, another prospective teacher, PMT37, gave the following answer to the open-ended question:

> *I would bring a rectangular prism box and unit cubes to class. I would ask the class to determine the volume of the box. During this process, we can introduce the concept of placing the unit cubes inside the box and encourage them to adopt the idea that the box is full. Or the misconception can be eliminated by showing the location of the unit cubes in the volume of the rectangular prism using GeoGebra software.*

In the scripting task application, PMT37 advanced the provided scenario in the following manner:

> ***Teacher: How did you calculate the volume? Who wants to explain?***
>
> ***……………………………………………………………………………………***
>
> *Teacher: Could you proceed to color each of the cubes you calculated one by one?*
>
> *Nazlı: I marked 6 cubes on the side and 12 cubes on the front. I marked 6 cubes. On the top side, I marked eight cubes. I also marked the other sides.*
>
> *Teacher: Let's look at the marks on the cubes.*
>
> *Nazlı: Some cubes had three marks and others had two marks. So, I overcounted.*
>
> *Teacher: Yes, what should you do with this?*
>
> *Nazlı: There are four cubes, with a height of three cubes and a width of two cubes. 4 x 3 x 2 = 24 cubes. Thus, the volume is 24.*

PMT37 proceeded in parallel with her first answer in the scenario, coloring and marking the concrete material. She made the following statement as a justification for her scenario: "*The student knows that the concept of volume is the total number of cubes, but it was explained that she overcounted the cubes while calculating and what the correct answer is*."

PMT25 is one of the PMTs who focused on conceptual understanding in theory and approximation of practice, but did so without using concrete models and technology. PMT25's answer to the open-ended question is as below:

> *After identifying the front surface, teachers can ask the student to count the number of identical shapes made up of 20-unit cubes, helping them understand concepts like depth and width. You can also apply this process to the side faces and base instead of the front face, ensuring the student understands the formula.*

PMT25's scenario is as follows:

> ***Teacher: How did you calculate the volume? Who wants to explain?***
>
> ***……………………………………………………………………………………***
>
> *Teacher: So, let's take turns finding the total number of cubes. How do we find the number of cubes in the base?*
>
> *Nazlı: There are eight pieces, each measuring 4 x 2.*
>
> *Teacher: What is our height?*
>
> *Nazlı: Since it consists of three cubes, the height is three, teacher.*
>
> *Teacher: How many cubes do we get when we stack these three more times, so that there are eight cubes at the base?*
>
> *Nazlı: We say that 8 + 8 + 8 = 24, or 3 x 8 = 24.*
>
> *Teacher: What type of operation did we perform with the numbers in this example?*

Nazlı: First, we made 4 x 2, and then we multiplied this result by 3. We calculated the result as (4 x 2) x 3.
Teacher: Then how do we calculate the volume and get the result?
Nazlı: We can quickly calculate the volume by multiplying the lengths of the segments.

PMT25 justified her scenario by stating, "*When determining the total number of cubes in the volume calculation, we followed the correct instructions. We helped the student understand the elements she utilized.*"

PMT10 is one of the PMTs who follows a procedural focus in both theory and practice approximation. In response to the open-ended question, PMT10 suggests that "*If she chooses this approach, should refrain from counting the cubes she initially counted*."

PMT10's scenario also supports this view:

Teacher: How did you calculate the volume? Who wants to explain?
...
Teacher: No, Nazlı. You found the surface area of a rectangular prism, not its volume. When we find the volume of a prism, we do it as the base area times the height.
Nazlı: I understand, teacher. Then the base is 4 x 2 = 8, and the height is 3. Therefore, 8 x 3 equals 24 cubic units.
Teacher: Yes, Nazlı, very good.

The justification for PMT10's scenario is as follows: "*Since the student confused the volume with the surface area, I told the correct method to correct what she knew wrong.*"

Based on PMT10's response to the open-ended question and the completed scenario, it is evident that she implemented instructional strategies such as directly addressing the student's mistake, adhering to a procedural focus, and waiting for the student to clarify by providing the formula.

As another example, PMT27's response to the open-ended question is "*First, I explain the difference between volume and area, and then I explain the volume of a rectangular prism.".* Her scenario and justification are below respectively:

Teacher: How did you calculate the volume? Who wants to explain?
...
Teacher: Nazlı, you thought right. It is enough to find the total number of cubes. However, you counted more than one cube in the last operation.
Nazlı: What do you mean, teacher?
Teacher: On the top side, there are eight cubes. Similarly, there are two sets of eight cubes underneath. This way, we count them once. So, 8 x 3 = 24. Alternatively, you can use the formula

to calculate the volume of a rectangular prism more easily. Length x width x height. 4 x 2 x 3 = 24 cubic units.

"I moved the conversation along in this manner because, although the student had good reasoning, she was unable to put it into practice when performing the task."

The prospective teacher in this example favored a procedural focus, which she reflected in her approximation of practice.

Inconsistency of Theory-Approximation of Practice

To exemplify the PMTs who aimed to teach conceptual understanding in theory but could not reflect this in approximation of practice, PMT29 shared the following statement in response to the open-ended question: "*The student is asked to form this prism by giving unit cubes and then asked to calculate its volume*." To complete the approximation of practice, PMT29 followed the steps in her scenario listed below:

Teacher: How did you calculate the volume? Who wants to explain?

...

Teacher: Don't the numbers you found correspond to the number of squares associated with that face?

Nazlı: No, the number of cubes.

Teacher: Don't the faces of a cube belong to both the side faces and the top face?

Nazlı: Yes.

Teacher: Then, don't four of the 12 cubes on the front face also belong to the top face?

Nazlı: Yes, what I found was the area, not the volume. The correct answer is 4 x 3 x 2 = 24 cubic units.

PMT29 shared the following statement as a justification for her scenario: "*I advanced the scenario in this way because the student calculated area, not volume.*"

From PMT29's scenario and the explanation she offered, it's clear that while her goal was to help the student advance through the use of concrete materials in theory, she didn't incorporate this into the approximation of practice. Instead, she used questions to help the student understand that she found surface area, not volume. As a result, the student had a rapid enlightenment and calculated the volume using the formula.

Similarly, PMT21 responded to the open-ended question by stating, "*First and foremost, I would focus on determining the volume of prisms. We can include studies using prism models to enhance conceptual understanding.*"

PMT21's scenario is as follows:

Teacher: How did you calculate the volume? Who wants to explain?

………………………………………………………………………………………

Teacher: Okay, Nazlı, let's find the total number of cubes together. On the bottom base, there are four cubes. There are two rows of cubes on the side and three more rows of cubes on the top. So, in total, there are 4 x 3 x 2 cubes. So, there are 24 cubes.

Nazlı: Okay, teacher. If there are 24 cubes, then the volume is 24.

Teacher: Yes, Nazlı, since there are 24 cubes, the volume will be 24.

Examining PMT21's justification revealed that she made the following claim: "*The student found it incorrect because she calculated the side surface areas instead of volume. To remedy this, the student should count the cubes and then switch to volume.*"

It is clear that PMT21 aimed to advance by using the model in theory; however, she was unable to do so in the approximation of practice and instead chose to explain the solution.

DISCUSSION

The present study aimed to explore the alignment between PMTs' skills in responding to student thinking in theory and approximation of practice in the context of conceptual and procedural understanding. The findings primarily revealed that 68% of PMTs planned to respond to student thinking in theory by using concrete models and technology, while only 18% planned to directly provide the answer to the student and instruct the student to perform the operation using a formula. On the other hand, only 32% of PMTs were able to approximate practice using concrete materials and technology, while 50% of PMTs provided the answer or formula to the student. Analyzing the alignment between theory and approximation of practice revealed that less than half of the PMTs (45%) aimed to teach for conceptual understanding, and were able to demonstrate this in scripting task application. 31% of PMTs aimed for conceptual teaching but could not reflect this in their approximation of practices. The PMTs who followed procedural focus in both theory and approximation of practice also showed a similar rate (24%).

As mentioned before, teachers are expected to build learning environments where students are at the centre and conceptual understanding is prioritised (MoNE, 2018). For this purpose, in teacher training programmes, prospective teachers are trained in the context of planning and implementing lessons in which students are placed at the centre through the use of multiple representations and in-class discussions in line with the constructivist approach. At this point, the fact that the findings of the study revealed that the teaching idealised by prospective teachers in theory was predominantly student-centred and aimed at conceptual understanding indicates that undergraduate education can achieve its purpose up to a certain level. On the other hand, it is noteworthy that this goal, which can be achieved in theory, is not sufficiently reflected in practice.

Schoenfeld (2011) underlines that there is a consistently large difference between the number of teachers who express ideas about student-centered teaching and those who actually implement these beliefs. Being in parallel, the findings of the present study support the findings of Osmanoglu and Girit-Yildiz's (2024) study, which showed that prospective mathematics teachers mostly idealized teaching reform-based lessons, but their responding in approximation of practices did not match this idealization. In the aforementioned study, while prospective mathematics teachers put forward reform-compatible instructional visions, they mostly followed traditional methods in their instructional approximation of practices examined through the scripting tasks. Accordingly, the PMTs may exhibit more of a procedural approach in their approximation of practices, give the answer directly instead of letting the students reach the answer themselves, prevent them from having meaningful learning opportunities with overly directive questions, and expect the students to have a rapid enlightenment. Son's (2013) study also reported these findings. According to Son (2013), prospective teachers may focus on procedural focus while responding to students' misconceptions in a way that does not coincide with their instructional goals. Other studies in the literature (e.g., Guillaume & Kirtman, 2010; Shure & Liljedahl, 2023) also revealed similar findings. As Shure and Liljedahl (2023) stated, prospective teachers may have difficulty responding to student thinking and may move away from the focus on conceptual understanding. Beyond studies with prospective teachers, even studies conducted with in-service teachers may reveal that teachers are unable to respond effectively to student thinking. For example, according to a study by Kabadaş and Yavuz Mumcu (2024), teachers frequently dealt with students' errors superficially and failed to pay enough attention to their thinking when attempting to identify students' misconceptions. When attempting to eliminate the misconceptions, they informed students that their responses were incorrect and were unable to help them recognize their own errors.

One of the possible reasons underlying the fact that prospective teachers exhibit an understanding that is compatible with the reform and centered on meaningful learning in theory but fail to realize this at the expected level in approximation of practice may be that they have been predominantly exposed to traditional teaching in the education and training processes they have experienced until their undergraduate education (Anderson et al., 2005). During their four-year undergraduate education process, prospective teachers receive training on the importance and necessity of teaching with a constructivist and student-centered approach, which equips them with knowledge about the benefits of such an education and their plans to provide it. However, during the implementation phase, prospective teachers often struggle to translate this knowledge into practical applications. Factors such as a lack of teacher knowledge and experience may also play a role at this point. According to Schoenfeld (2011), resources such as knowledge and routines are effective for teachers to achieve their goals and may explain the mismatch between goals and approximations of practices. On the other hand, we understand that prospective teachers require support to reflect their ideals into their approximation of practices, regardless of the possible reasons. At this point, considering that prospective teachers had the chance to concentrate on student thinking through scripting tasks (Grossman et al., 2009), we believe that integrating pedagogies of practice opportunities, like scripting tasks, into the initial teacher education process could help future teachers better integrate their ideals into their engagement with approximation of practice.

CONCLUSION and SUGGESTIONS

Prospective teachers require a variety of professional development opportunities to enhance their capacity to respond to student thought processes. We can overcome the incompatibility between theory and approximation of practice by providing prospective teachers with the experience of thinking, reasoning, and responding to student thinking through applications such as scripting tasks. As stated before, these tasks serve as a simulation of classroom approximation of practices in teacher training programs, and are accepted as a representation and approximation of practice tools. In such development environments, we should first expect prospective teachers to overcome any conceptual understanding deficiencies they may have, and then focus on conceptual understanding to support student learning in the context of responding to student thinking (Shure & Liljedahl, 2023). At this point, in future studies, it may be recommended to examine the conceptual understanding deficiencies of prospective teachers through the scripting task applications and to provide professional development environments in order to overcome them. In these studies, prospective teachers might be provided with feedback regarding their theory and approximation of practice alignment with related suggestions to improve the consistency in line with reform. More detailed findings can be obtained by applying more scripting tasks. One-on-one interviews with prospective teachers can be used to enrich the data.

ETHICAL TEXT

"This article complies with the journal's writing rules, publication principles, research and publication ethics rules, and journal ethics rules. The responsibility for any violations that may arise regarding the article belongs to the authors. The ethics committee permission of the article was obtained by Trakya University Social and Human Sciences Research Ethics Committee with the decision numbered E-29563864-050.04.04.04-368280 dated 07.12.2022."

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